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VERIFICATION OF RAIN-FLOW RECONSTRUCTIONS OF A VARIABLE AMPLITUDE LOAD HISTORY

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VERIFICATION OF RAIN-FLOW RECONSTRUCTIONS OF A VARIABLE AMPLITUDE LOAD HISTORY

by

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ABSTRACT

The suitability of using rain-flow reconstructions as an alternative to an original loading spectrum for component fatigue life testing is investigated. A modified helicopter maneuver history is used for the rain-flow cycle counting and history regenerations.

Experimental testing on a notched test specimen over a wide range of loads produces similar lives for the original history and the reconstructions. The test lives also agree with a simplified local strain analysis performed on the specimen utilizing the rain-flow cycle count. The rain-flow reconstruction technique is shown to be a viable test spectrum alternative to storing the complete original load history, especially in saving computer storage space and processing time.

A description of the regeneration method, the simplified life prediction analysis, and the experimental methods are included in the investigation.

(Based on the M.S. Thesis of John D. Clothiaux, May 1990)

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List of Symbols

S	Nominal component stress
N	Component life
K	Stress intensity
k_t	Elastic stress concentration factor
k_σ	Local stress concentration factor
k_ϵ	Local strain concentration factor
σ	Notch stress
ϵ	Notch strain
E	Elastic modulus
A	Cyclic strength coefficient
s	Cyclic strain hardening exponent
n	Number of cycles at a specified value
B	Number of block repetitions of history until failure
ϵ_a	Strain amplitude at notch
N^*	Cycles to failure for fully reversed cycles
σ_f'	Fatigue strength coefficient
b	Fatigue strength exponent
ϵ_f'	Fatigue ductility coefficient
c	Fatigue ductility exponent
σ_o	Mean stress for a given cycle
F	Dimensionless geometry factor
a	Crack length
C	Crack growth rate coefficient
m	Crack growth rate exponent
r_o	Cyclic plastic zone size
R	Ratio of minimum and maximum values in a cycle

γ	Walker coefficient
ΔK	Stress intensity range for one cycle
$\bar{\Delta K}$	Equivalent zero-to-maximum stress intensity for one cycle
ΔK_{eq}	Equivalent zero-to-maximum stress intensity for a block of cycles
N_b	Number of cycles in a block
a_i	Initial crack length
a_f	Final crack length
N_g	Total number of cycles during crack growth
B_g	Total number of blocks during crack growth
B_i	Total number of blocks during crack initiation
B	Total number of blocks during specimen life
r	Notch radius
c	Plastic zone size
l_m	Transition crack length for crack growth
P	Component load
P	Normalized component load

1.0 Introduction

Improvement of the fatigue life of a component in service is a goal of mechanical design. Since the component is typically subject to variable amplitude loading, accurate life prediction and appropriate testing are necessary to better the design process. This study applies two methods based on rain-flow cycle counting to these areas. A helicopter loading spectrum is the history to which the techniques are applied.

Accurate life prediction depends on analysis that sensibly models the behavior of the component. Three techniques are widely used for predicting component life: stress-life (S-N) curves, local strain approach, and fracture mechanics. The S-N approach typically correlates nominal component stress to a damage fraction for a stress cycle and consequently applies a damage summation rule for all the cycles to obtain specimen life to failure. The local strain approach, also based on damage accumulation, models material behavior at specific points in the component, such as a notch, to predict the life for a crack formation in the component. Fracture mechanics, based on linear elastic behavior, models crack growth in a component assuming some small initial crack size. The method used in the prediction of life for the helicopter spectrum is a combination of the local strain approach and fracture mechanics.

For experimental testing, obtaining concise representations of field service loads is desirable. Statistical descriptions, such as power spectral density functions and deterministic descriptions, such as cycle counting, prevail in this area. Both methods reduce field data into a compact form that is used to reconstruct loading spectra suitable for testing and to provide input for theoretical life predictions.

The rain-flow cycle counting technique suits both tasks well. When coupled with the local strain approach, a prediction of the life to crack initiation is allowable. In addition, the rain-flow cycle count provides a convenient basis for reconstructing load histories suitable for component testing. These two features imply that the rain-flow description is an appropriate choice for fatigue analysis.

To this end, modeling developed by A. K. Khosrovaneh at VPI based on the rain-flow description is experimentally verified. Two specific areas are examined for the helicopter load spectrum applied to a notched specimen: a simplified analysis of life prediction based on the local strain approach and history regeneration from the rain-flow cycle count. The life equivalency of different histories based on the rain-flow cycle count and the accuracy of the simplified analysis are the two principal points of investigation.

2.0 Review of Literature

2.1 Types of Loading

Service loads on a structure and its individual components are needed to perform realistic fatigue calculations and tests. Unfortunately, the complete loading characteristics cannot be measured directly. However, strains at specific points and interfacing loads can be measured with strain gages, load cells, and a variety of other methods [1]. These load data may then be extrapolated into a full stress field for the component of interest. The variation of this stress field over time dictates the fatigue life for a given component. Therefore, collection of the loading data used in modeling must contain all possible loading cases to ensure accurate representation of the actual service stresses [2].

Component loading in service comes from three sources: random vibrations, duty cycles, and rare events. Random vibrations typically result from structural vibration and environmental conditions, such as wind gusts and road roughness. Duty cycles arise from the designed service usage of a component, such as an automobile steering column subject to turning. Rare events are unexpected occurrences, such as striking a large pothole. But if all events can be accounted for sufficiently and completely, the measured load history may be used with confidence in analysis and testing [2].

2.2 Characterizing Load Histories

The two fundamental approaches to characterizing measured load histories are statistical methods and deterministic methods. In statistical methods, the load history is modeled with a power spectral density function derived from Fourier analysis of the load-time history. This type of modeling depends on the load history being random. Hence, it is ideal for describing the vibrational portion of the load history [3–6]. However, if deterministic mean load variations occur in the history, statistical methods tend to produce erroneous results.

Deterministic methods are based on cycle counting. Level crossing counts, peak counts, and hysteresis loop counts are the three broad categories of counting[7]. As seen in figure 1, a variety of sophistication levels apply for the different methods which are described in reference 8. The simpler cycle counting methods, though easier to perform, may sometimes lead to poor modeling[9,10]. Therefore care must be exercised in choosing a cycle counting technique. The rain-flow method counts cycles based on the hysteresis loops in the material. This natural cycle definition makes it widely accepted as the best method for deterministic cycle counting [10–16].

2.3 Load History Reconstruction

Both statistical and deterministic characterizations of load histories may be used to regenerate loading histories. Reconstruction techniques that create the exact original history from some compact characterization have not been achieved due to complex algorithms and large storage requirements[16]. Therefore, reproduction of histories that yield damage equivalent to the original history, without necessarily being identical to the original history, is desirable [2].

Reconstructions from statistical methods produce good results for random histories. Two typical techniques are the summation of spectral components [4] and to-from reconstruction based on a Markov process assumption [16]. But the helicopter load spectrum used in this study contains definitive mean shifts [17], so the applicability of these methods is poor. Therefore, reconstructions based on cycle counting must be employed.

Reconstructions from simple cycle counting methods may produce histories with damage characteristics different from the original history. This problem arises from shortcomings in the cycle counting techniques [10]. The simple counting methods may miss mean level variations, causing too conservative a history, or produce too many large cycles from a reordering of the peaks. Reconstructions from the rain-flow technique, however, always produce the same load cycles. Only the ordering of the cycles is different.

The development of the rain-flow regeneration technique is limited to a few workers [12–14]. Perret has published the only data specifically comparing different experimental variable amplitude history lives based on the same rain-flow cycle count [12]. The tests were performed at one stress level on notched aluminum specimens, comparing the crack initiation and crack growth lives for different *structured* regenerations of the standard loading spectrum FALSTAFF. The results supported the validity of the rain-flow reconstruction technique. The reconstruction technique developed by Khosrovaneh [14] creates *random* regenerations and will thus be applied to the helicopter load spectrum and tested to expand the verification of this method.

2.4 Predicting Component Life

Life prediction and cycle counting are independent of each other. Any pairing of the two procedures will yield a life prediction because any life prediction model requires cycle information for input. Hence the fatigue life may be predicted once the cycle counting is performed. Since the rain-flow cycle counting procedure is used for load spectrum regeneration, it is also used for input into the life prediction model, as a matter of convenience. This is desirable, however, for the rain-flow method typically predicts life better than the simpler cycle counting methods [9]. The appropriate choice of life prediction model to couple with the rain-flow cycle counting must now be made.

The three techniques commonly used for life prediction are the stress-life (S-N) approach, the local strain approach, and linear elastic fracture mechanics [9]. The first two techniques are similar in that they compute the damage done by each cycle based on data gathered from constant

amplitude tests of the same material. Then the cumulative damage for all the cycles is combined via a damage accumulation rule.

The S-N approach [18] is based on data from constant amplitude stress-life tests. The data from these tests are used to form a power equation or other relationship that defines the life for every stress range. Empirical adjustments to this curve are then required for any deviations from the specimen geometry or from the reversed loading for which the stress-life relationship was established. Selection of the constants to make these adjustments can be inexact due to their empirical relationship. Therefore, the selection of this method should be limited to components where stress-life data exist, or to situations where more specific modeling is difficult, such as welded components. This method predicts the total life of a specimen.

The local strain approach, which predicts only the crack initiation life, improves the S-N approach by modeling the specific material behavior at a point of interest, like a notch. The stress and strain variations for each load variation are computed and related to life. The local strain approach employs data from constant amplitude strain-life tests. However, it also adjusts the predicted damage due to the mean stress present for each strain cycle [19-21]. The mean stress depends on the cycle order, so obtaining an exact prediction for the life requires consideration of the entire history and not just the rain-flow matrix. However, a simplified analysis described by Dowling and Khosrovaneh [22] employs the local strain approach to predict bounds on the specimen life by considering only the rain-flow count. This method greatly reduces the computational effort as compared to the full local strain analysis, which must follow the notch response for every load reversal [23].

On the other hand, fracture mechanics depends on quantifying a specific relationship between the crack growth rate and the stress intensity [24]. The value of the stress intensity, K , combines the severity of the loading, crack size, and specimen geometry [25]. The inverse of this relationship is then integrated over the range of crack lengths to produce a crack growth life [26]. However, the life predicted by this method is very sensitive to the initial crack length. So designing components based on crack growth lives requires one of two necessities: reliable detection of a specified minimum crack size, allowing components to be designed that can sustain cracks smaller

than this crack size, or component design which can sustain cracks for a long time. Many components cannot meet either of these requirements [27], so fracture mechanics is unsuitable in these cases.

Each of the above techniques is applicable in certain situations. However, if a component is designed such that significant portions of its life are spent in both crack initiation and crack growth, then a combination of both the local strain approach and fracture mechanics is suitable for predicting life [28]. This combined technique for predicting life is applied to the life prediction of the test specimen of this study.

3.0 Fatigue Life Methodology

The rain–flow cycle counting method is the basis for life prediction and history regeneration. Therefore, the rain–flow cycle counting method is discussed first. Then, life prediction using the simplified analysis is explained. The simplified analysis, derived from the local strain approach employed for predicting crack initiation, is based on the Palmgren–Miner rule for damage accumulation [14]. Application of the Palmgren–Miner rule requires consideration of three areas of the analysis: appropriate cycle counting, local notch plasticity altering the mean stress, and periodic overstrains changing the life fraction due the smaller cycle ranges [10,15]. Finally, history reconstruction based on the rain–flow cycle counting is discussed.

3.1 Rain–Flow Cycle Counting

The loading history utilized in the life predictions and experimental testing is for the tail rotor pitch beam of an AUH–76 helicopter. The loading history is a combination of 30 sequences derived from distinct, severe maneuvers. The original history of 33,470 cycles was modified by the University of Dayton Research Institute and subsequently filtered and normalized by Khosrovaneh [14] to produce a history of 510 cycles, or 1020 load reversals, as shown in figure 2. This history is referenced throughout this thesis as the filtered maneuver history. For convenience, one repetition of this history is called a flight.

The rain–flow cycle counting method is applied to this helicopter history in order to perform the simplified life analysis and to reconstruct different histories for testing. This section presents a

brief description of the rain-flow cycle counting method. The complete set of rules for performing rain-flow cycle counting is described in references 8 and 14.

For a repeating history, like the filtered maneuver history, the order of peaks may be arranged so that the largest peak is first in the history, as shown for a simple history in figure 3. This re-ordering facilitates the rain-flow cycle counting. When a load history is applied to a notched specimen, any pair of load reversals that forms a closed stress-strain hysteresis loop in the material at the notch is considered a cycle, for example, H-A and C-D in the simple history of figure 4. While performing the rain-flow cycle count, each time a cycle is counted, the appropriate peaks are removed from the history and the extracted cycle is recorded. This method is shown in figure 5 for the simple history. The extracted cycles may be presented in tabular form (figure 5e), showing the exact values of the cycle starting and ending loads or the cycle range and mean load.

However, if the history contains many load reversals, a table showing all the cycles is quite long. This size may be reduced if the history is discretized into load levels, typically 32, as shown for the simple history in figure 6a (using eight load levels for illustrative purposes.) The load levels are determined by dividing the difference of the maximum and minimum load values by the number of load levels less one. Each load reversal is rounded off to the nearest load level during the counting procedure, so the cycles are defined by pairs of load levels instead of exact load values. The resulting cycles may then be compactly stored in a matrix whose row holds the starting level and column holds the ending level. If the direction of the cycle is stored, then the entire matrix is used (figure 6b), whereas if the cycle direction is not considered, only the half below the diagonal is needed (figure 6c.) In addition, a matrix storing the cycle range and mean could be used. But the matrix containing the starting load levels and ending load levels is more suitable for performing history reconstructions, so this form is used in the study.

The rain-flow cycle counting is performed on the filtered maneuver history, using the computer program explained in section 4.1. The resulting matrix, considering cycle directions, is shown in figure 7. This matrix provides the basis for the life prediction calculation and history reconstructions to be explained next.

3.2 Life Prediction

When a component is subject to a variable amplitude load, the material around the notch may yield, due to the stress concentration of the notch, while the overall component behavior remains elastic. This yielding produces a redistribution of stress in the vicinity of the notch (figure 8) and may profoundly affect the actual behavior of the yielded material. In fact, the first step in analyzing the notch material is to relate the stress-strain behavior at the notch with the nominal component stress variation. The analysis assumes that the material at the notch is in plane stress, allowing its behavior to be predicted directly by material constants determined from testing on smooth axial specimens [29]. Then, from the actual behavior of the notch material, life may be predicted from the P-M rule using rain-flow cycle counting. Again, care must be used when accounting for mean stress effects and the overstrain effect.

3.2.1 Notch Plasticity

Quantitative methods can be applied to relate the stress-strain behavior at the notch to the far field stress. Nonlinear elastic-plastic mathematical models can be implemented or strain gages can be mounted to experimentally determine this relationship. However, these analyses are expensive to perform and must be done on every geometry of interest [30]. Another technique is to apply Neuber's rule for a notch [31]. Neuber's rule states that the geometric mean of the local stress and strain concentration factors is equal to the elastic stress concentration factor,

$$\sqrt{k_\sigma k_\epsilon} = k_t. \quad (3.1)$$

Assuming that the component away from the notch remains elastic in stress and strain, equation 3.1 reduces to [31]

$$\sigma \epsilon = \frac{(k_t S)^2}{E}. \quad (3.2)$$

Equation 3.2 allows the product of the stress and strain at the notch to be computed in terms of the remote stress. A second equation is now needed to uniquely evaluate the stress and strain. The stress-strain equation is an equation based on constant amplitude strain tests that relates the strain amplitude to the stress amplitude for a given material. The equation usually takes the form [32]

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{A} \right)^{\frac{1}{s}} \quad (3.3)$$

The constants A and s for the cyclic stress-strain equation may differ from the monotonic values, due to cyclic softening or hardening, but the cyclic values for the component material must be known in order to compute the cyclic stress and strain. At any rate, the cyclic stress-strain equation (equation 3.3) and Neuber's rule (equation 3.2) are solved together to determine the values of the notch stress and strain as a function of the remote stress.

For the notch analysis, the calculation of the notch stress and strain actually takes two distinct forms. First, the maximum values of the notch stress and strain are determined from the maximum nominal load in the history by employing equations 3.2 and 3.3. Then, the equations are used to relate the stress and strain amplitudes with the nominal stress amplitude, effectively doubling the size of the cyclic stress-strain curve, as illustrated in figure 9 for three load reversals. This relationship forms the model of the hysteresis loop curves describing the stress-strain response. From these curves, the values of the stress amplitude and strain amplitude are known for each load cycle of a component. These values are then used to determine component life as described in the next section.

3.2.2 Damage Accumulation

The Palmgren-Miner rule (P-M rule) is the traditional technique used for accounting the life fraction of a given stress or strain cycle on a component [33,34]. For the S-N approach, the life fraction is based on the stress amplitude, hence the life prediction is based on the stress amplitude of the material response. However, for a notched specimen, the local stress at the notch may not behave in the same manner as the general behavior of the component. As shown in figure 10, the behavior of

the stress and strain depends on the order in which the loading occurs. Following the order of cycles is important in determining the exact stress-strain response for predicting life. Hence for any type of notched specimen, the life fraction should be calculated by the strain amplitude at the notch, with adjustments being made for the mean stress present for each strain cycle.

The P-M rule states that the life fraction from a given strain cycle is the equivalent fully reversed cycle range divided by the total life of the material at that range. Failure is reached when the sum of life fractions equal one,

$$\sum \frac{n_i}{N_i} = 1 \quad (3.4)$$

If the cycles are part of a repeating block, then the number of block repetitions is

$$B \sum \frac{n_i}{N_i} = 1 \quad (3.5)$$

The relationship between strain amplitude and total life is experimentally determined from tests using constant amplitude fully reversed strain cycles. The data from these tests are used to establish the strain-life equation [35],

$$\epsilon_a = \frac{\sigma_f'}{E} (2N^*)^b + \epsilon_f' (2N^*)^c \quad (3.6)$$

for the component material. An adjustment must be made for the mean stress of each actual strain cycle to produce the equivalent fully reversed strain amplitude in order to utilize the strain-life equation.

Two methods are generally accepted for accounting for the mean stress, the method of Morrow [36] and the method of Smith, Watson, and Topper [37]. For this analysis, the Morrow rule is used. The mean stress will change the predicted life of a fully reversed strain cycle by an amount

$$N = N^* \left(1 - \frac{\sigma_o}{\sigma_f'} \right)^{-\frac{1}{b}} \quad (3.7)$$

This adjusted total life is then used to compute the life fraction of the strain cycle. As equation 3.7 indicates, noting that b is negative, if the mean stress is greater than zero, then the damage done by a

given strain amplitude is greater than the same strain amplitude with no mean stress. So the mean stress must be used to adjust the life fraction of each strain cycle to prevent non-conservative life predictions. By modifying the total predicted life for each strain cycle going into the P-M rule, a more accurate prediction of life is expected. A summary of these adjustments is given in figure 11.

The above procedure outlines the local strain approach based on constant amplitude strain-life data. It is used to predict the number of blocks, B , of a repeating history that a notched specimen will endure until crack initiation. The analysis incorporates the rain-flow cycle counting method to determine nominal stress cycles and accounts for the mean stress of each strain cycle. However, the overstrain effect can also alter the life prediction.

3.2.3 Overstrain Effect

The overstrain effect is the third consideration in applying the P-M rule. This effect does not change the procedure of the local strain analysis. Rather, the effect alters the life fraction predictions for the lower strain amplitudes. The effect is rooted in the manner in which the life fractions are summed [9]. To simply add all the life fractions to predict life assumes that the physical damage due to each cycle is uniquely related to the life fraction [38]. However, if the relationship varies for different levels, failure may be reached before the sum of life fractions reaches one. This difference is shown in figure 12.

Normally in a strain-life test the strain amplitude is held constant for the entire test. But if a periodic overstrain is introduced in the longer lived tests, the life of the test is reduced [39–41]. The overstrain introduced into the constant amplitude tests does not significantly contribute to the damage as calculated by the P-M rule. Instead, introducing the large strain causes the lower strain amplitudes to effectively do more damage, hence reducing the life. Neglecting this behavior can produce an overestimate of component life. The effect can be seen in a change in the strain-life curve for aluminum (figure 13a.) In addition, for steels, the overstrain may eliminate the endurance limit exhibited in low level constant amplitude stress tests (figure 13b.) To incorporate this effect into the analysis requires the longer lived constant amplitude strain tests to contain periodic overstrain.

The results from these tests will affect the lives in the regime of lower strain amplitudes. The exact nature of the overstrain effect on the 4340 steel used in the testing can be incorporated into the analysis, either by producing a new set of constants for the strain-life equation or by eliminating the endurance limit, or both. The end result is probably a more conservative prediction of the life of a notched component. However, only experimental testing yields the severity of the overstrain effect on the component material used in this study.

3.3 Simplified Life Analysis

A technique described by Dowling and Khosrovaneh [22] can be used to predict the life of a notched component under variable amplitude loading from the rain-flow matrix. Since the rain-flow count loses information about the ordering of cycles, and hence the local notch mean stresses, the technique does not predict a single component life. Instead, the analysis calculates the maximum and minimum possible mean stresses for all the cycles and uses those values to place bounds on the life of a repeating block.

For a given cycle in the rain-flow count, the bounds on the local notch mean stress are determined by the relationship between the stress-strain hysteresis loop formed by that cycle and the hysteresis loop formed by the largest cycle in the history. The loop for any cycle must be contained within the loop for the cycle defined by the extreme values. This restriction is illustrated for cycle H-A of the simple history of figure 14.

The values of the nominal stress are fixed by the cycle definition and the nominal stress-strain response for any cycle is fixed within the largest loop as shown in figure 14a. These two extreme positions define the maximum and minimum possible initial strain values for the cycle. From these imposed strain values, the notch stress-strain loops are hence defined as shown in figure 14b. From the local notch stress-strain response, the extreme values of the mean stress for the cycle are determined. Finally, the maximum and minimum life fractions are determined based on the equivalent fully reversed strain amplitude, discussed in section 3.2.2.

These bounds may be placed on every cycle in the rain-flow matrix in a similar manner. The simplified analysis combines all the life fractions due to the lowest mean stress for each cycle into a prediction of a maximum life for the history. Additionally, all the life fractions are calculated for the maximum mean stress of every cycle resulting in a prediction for the minimum life. Since these bounds are determined from a rain-flow matrix, any load spectrum with that same rain-flow count should ideally yield a life between these bounds when tested on a component.

Very little experimentation has been done comparing the lives of different histories with identical rain-flow cycle counts. Perrett [12] compared lives of histories that were deterministically derived from a rain-flow cycle count. That is, the cycles were placed in a particular order to bias the testing life of the specimen. The results from the testing showed little difference in life between the histories, even though the histories were created to maximize the sequence effects.

The object here, however, is to compare histories of the same rain-flow count that are reconstructed in a random fashion and to compare the histories to the bounds predicted by the local strain analysis. Chapter four discusses the computations necessary to perform a rain-flow cycle count, to predict life with a simplified local strain analysis, and to create randomized regenerations from the rain-flow matrix. The reconstructions are compared experimentally for crack initiation, and they are also compared for total specimen life, which includes crack initiation and crack growth. Therefore, fracture mechanics for crack growth needs to be reviewed.

3.4 Crack Growth

The primary emphasis of this study is to compare the life of reconstructed histories to the original history and to the bounds predicted by the simplified analysis. A crack growth analysis is included for two reasons. One reason is to help clarify the definition of crack initiation, and the other is to predict total specimen life. The total life prediction enables the notched specimen lives to be compared to a predicted life as well as to each other. The total life is the sum of crack initiation life plus crack growth life. The local strain analysis is used to predict crack initiation life. Now crack growth is discussed to complete the total life prediction.

The effect of a crack in a specimen may be described in linear elastic fracture mechanics (LEFM) by the stress intensity, K . K is dependent on the size of the crack, the geometry of the cracked body, and the nominal stress field in the vicinity of the crack. All the effects combine to form one equation [25],

$$K = FS\sqrt{\pi a} \quad (3.8)$$

where F is a dimensionless function of crack length and specimen dimensions, S is the nominal stress, and a is the crack length. Since the stress intensity depends on the nominal stress, and hence the load, any load variation will cause a stress intensity variation. Crack growth is based on this variation in stress intensity.

Crack growth for a zero to maximum load application to a cracked member is described by the Paris equation [24] which relates the crack growth rate to the stress intensity range,

$$\frac{da}{dN} = C\Delta K^m, \quad (3.9)$$

provided the plastic zone at the leading edge of the crack is small compared to the other dimensions of the member. The plastic zone size for cyclic loading is estimated from Irwin's equation [42],

$$r_{ps} = \frac{1}{\pi} \left(\frac{\Delta K}{2\sigma_o} \right)^2 \quad (3.10)$$

for a plane stress condition in the member. If the specimen satisfies the plastic zone constraint, then linear elastic fracture mechanics is applicable, and the above equations apply.

When ΔK is not zero to maximum, modifications to the stress intensity are necessary to obtain the correct crack growth rate. The Walker equation [43] adjusts equation 2.9 by calculating an equivalent ΔK ,

$$\overline{\Delta K} = K_{\max}(1-R)^\gamma \quad (3.11)$$

where γ is the Walker coefficient, a material property, and

$$R = \frac{K_{\min}}{K_{\max}}. \quad (3.12)$$

If R is negative, that is K_{min} is less than zero, then an additional assumption is made that no crack growth occurs during the compressive portion of the loading. Hence, for negative R ,

$$\overline{\Delta K} = K_{max} . \quad (3.13)$$

Obviously, if K_{max} is less than zero, no crack growth is expected to take place. Combining the appropriate equivalent ΔK with equation 3.9 allows calculation of the crack growth for any R -ratio.

For variable amplitude histories, the crack growth rate depends on the crack growth of many cycles with varying equivalent ΔK . For this situation, another equivalent ΔK must be calculated for the entire history [26]. If no interaction occurs between cycles, this is

$$\Delta K_{eq} = \left[\frac{\sum_{i=0}^N (\overline{\Delta K})^m}{N_b} \right]^{\frac{1}{m}} . \quad (3.14)$$

For a variable amplitude history of N_b cycles, the equivalent ΔK of equation 3.14 is expected to cause the same amount of crack growth over N_b zero to maximum cycles as the variable amplitude history. Once the crack growth rate is computed, the total crack growth life is found by integrating the inverse of the crack growth rate over the range of crack lengths [26], or

$$N_g = \int_{a_i}^{a_f} \left(\frac{dN}{da} \right) da . \quad (3.15)$$

Finally, the number of cycles of crack growth, N_g , is divided by the number of cycles in the history to yield the number of blocks to failure for the crack growth life,

$$B_g = \frac{N_g}{N_b} . \quad (3.16)$$

3.5 Total Specimen Life

The total life of a specimen during variable amplitude block loading consists of two phases. The first part of the total life, crack initiation, is predicted by the local strain analysis. Once the crack

has formed, LEFM is applied to crack growth, and the second part of the life is predicted. Then the total life of the specimen is predicted by the sum of the two phases,

$$B = B_i + B_g . \quad (3.17)$$

However, determining the transition point from crack initiation to crack growth requires some investigation [27].

The stress intensity, calculated by equation 3.8, does not follow the same character for all crack lengths in a specimen of plane stress with a circular hole. When the crack is short, the dimensions of the specimen are large compared to the crack length. This size disparity causes the stress intensity solution to act as if the crack on each side is comparable to an edge-cracked specimen (figure 15b). The resulting stress intensity is called the short crack case. But as the crack increases in length, the stress intensity solution begins to follow the solution for a center cracked specimen. When this transition occurs, the notch simply acts as part of the total crack length in the specimen (figure 15c.) Therefore, two distinct phases of the stress intensity are seen during crack growth [28], with a transitional crack length, l_m , being shown in figure 15d as the length corresponding to point P.

However, the stress intensity solutions are based on LEFM calculations. Some care must be exercised in dealing with notched specimens. In the LEFM discussion on crack growth, section 3.4, the plastic zone size associated with the crack must meet certain criteria for plane stress LEFM to be applicable. The notch also produces a plastic zone independent of the plastic zone associated with the crack. This plastic zone is examined to verify the applicability of the stress intensity solution.

For a notched specimen, when the nominal stress is such that yielding occurs at the notch, the stress at the notch must redistribute to accommodate the yielding. This phenomena leads to a plastic zone at the notch estimated by [28]

$$e = r \left[\left(\frac{k_t S}{\sigma_o} \right)^{\frac{2}{3}} - 1 \right] . \quad (3.18)$$

From equation 3.18, if $k_t S$ is only 30% larger than the yield strength, then the plastic zone size, e , is about one-fifth the notch radius, r . If the notch plastic zone size is comparable in size to the transitional crack length, l_m , then LEFM is not applicable to the short crack stress intensity portion of

the crack growth. To circumvent this conflict, the transitional length is used as the initial crack length for the crack growth [27]. This choice eliminates crack growth predictions made by LEFM for the short crack case from the crack growth life, yet it insures that the life prediction is not too conservative by capturing all of the long crack growth. This sensible choice of crack initiation length avoids use of much more complicated elastic-plastic crack growth computations in the short crack regime.

Using l_m as the definition of crack initiation finalizes the analysis. The local strain approach is applied to the test specimen geometry, using the appropriate material constants, to predict the number of blocks to crack initiation. LEFM crack growth is then applied, with l_m as the initial crack length, to predict the number of blocks during crack growth to failure. The failure crack length is determined by either gross yielding failure of the specimen or by the fracture toughness of the specimen being exceeded. Then the number of blocks from both stages are added to give the total predicted life. The computational procedure is the subject of the next chapter. However, the application of the rain-flow matrix for creating load histories is reviewed first.

3.6 History Reconstruction

History reconstruction is the second part of the application of rain-flow cycle counting. The rain-flow matrix is quite convenient for generating histories. The reconstruction process begins by placing the largest values of the rain-flow matrix into the history first and then continues by inserting increasingly smaller cycles into the larger ones until the entire matrix is exhausted. The resulting history will then contain the exact rain-flow cycle count as the matrix which produced it.

Figure 16a shows the insertion order that must be followed in an example eight level rain-flow matrix where cycle direction is considered. The largest cycles must be placed in the reconstruction first so that valid insertion positions are available for the smaller cycles. Placement of a smaller cycle with a larger one is shown in figure 16b for all the directional possibilities. By following the prescribed order, the inserted load reversals will always form a cycle in agreement with the rain-flow

cycle value. Note that every cycle for a given position in the rain-flow matrix must be inserted before moving to the next position.

Valid insertion sites are determined by the starting and ending levels for the cycle being inserted. For an ascending cycle, a valid position must contain a load reversal greater than or equal to the ending value of the inserting cycle, followed immediately by a load reversal less than or equal to the starting value of the inserting cycle. Examples of valid insertion sites in a short history for an ascending cycle are shown in figure 17. Similar rules apply to a descending cycle. A valid insertion site must contain a load reversal less than or equal to the ending value and be immediately followed by a load reversal greater than or equal to the starting value. Valid positions are shown in a short history for a descending cycle in figure 18.

Following the rules of cycle insertion will yield a regenerated history with the same rain-flow cycle count as the original history from which the matrix is created. However, some flexibility exists in the insertion site chosen for each cycle. As just shown for a simple example in figure 18, many valid locations are typically available for each inserted cycle. The choice of distribution of the inserting cycles will affect the final reconstructed history. For this study, the insertion site for each cycle is chosen at random, hence producing a different history each time the reconstruction is performed. Specific details about the computer program used to create the load spectra regenerations are discussed in section 4.4.

The two above applications of the rain-flow matrix, simplified analysis for life prediction and history regeneration, are the focus of experimental investigation in this study. The next chapter discusses the computer programs utilized to efficiently implement these techniques. Then experiments are performed to validate the life predictions and the test equivalency of the regenerated histories to the original filtered maneuver history.

4.0 Computer Analysis

A complete analysis of the filtered maneuver history acting on the notched test specimen is performed. FORTRAN programs developed by A. K. Khosrovaneh at VPI [14,44,45] are used to perform a rain-flow cycle count, predict the bounds of life using the simplified analysis, and reconstruct different histories from the rain-flow matrix. In addition, improvements to the reconstruction program and a crack growth analysis program are also utilized. Each program is discussed in the order of execution in the notched specimen analysis.

4.1 Rain-flow Cycle Counting

The first step in analyzing the filtered helicopter load spectrum is to perform a rain-flow cycle count using the program RAINF2. The program has several options for output, such as the form and size of the rain-flow matrix and the amount of individual cycle information. The option that creates a peak-valley count with consideration of cycle directions is used as the basis for this study. Another feature of the option is that the history is discretized into a specified number of rows and columns similar to section 3.6. For this study, the matrix was divided into 32 rows and 32 columns. A convenient result of the load discretization is an effective normalization that allows the rain-flow matrix to be easily scaled to different maximum values for experimental tests.

The resolution of the rain-flow count is perhaps too refined for a history of only 510 cycles. But applying this size matrix to a large history would not be unreasonable. And since the histories reconstructed from the matrix are experimentally tested, differences in life due to resolution are

minimized in the comparison of reconstructions. The rain-flow matrix output from this program is shown in figure 7.

4.2 Upper and Lower Bounds on Crack Initiation Life

Once the rain-flow cycle count is performed, the upper and lower bounds of specimen life for the modified history are calculated using the simplified local strain analysis program UPLO [44]. The analysis is performed on the notched rectangular specimen used in the experimental testing. The stress concentration factor is 3.92, and the specimen material properties are shown in table 1.

The chosen version of UPLO predicts specimen life based on a rain-flow matrix with the direction of the cycles considered, as in figure 6b. This version yields the same life as is obtained using a matrix without consideration of cycle direction, like figure 6c. However, since cycle directions are considered in the reconstructions, it is convenient to use the corresponding form of the matrix here as well.

UPLO requires more information than just the rain-flow matrix and specimen construction. Since the history is being applied to an actual component, realistic load ranges for application to the specimen are gleaned from the discretized rain-flow matrix. The maximum tensile loads input into UPLO range from 17.8 kN to 40 kN. Each load may be divided by the specimen cross-sectional area and multiplied by the stress concentration factor to produce the parameter, $k_t S_{max}$, the maximum notch stress from elastic analysis, which is actually input into UPLO. The selected range of loads produces maximum stress ranges, $k_t S_{max}$, of 1250 MPa to 2830 MPa. The lives predicted by UPLO range from 1.24 to 2340 flights. The bounds predicted for the entire range of $k_t S_{max}$ are shown in figure 19a. Now that the life of the specimen is predicted for an entire range of loads, experimental testing may be performed to compare different histories from the above rain-flow matrix against the lives to crack initiation predicted by the simplified analysis. Section 4.4 on rain-flow reconstructions explains the creation of the histories used for the testing. But the crack growth analysis is discussed first to complete the total life prediction for the specimen.

4.3 Crack Growth Life

The crack growth analysis is performed with FORTAN program CRACK. This program uses the rain–flow matrix output from RECON2 as input, along with material constants and initial and final crack lengths. The material constants for the crack growth equation are shown in table 1. Crack growth data from two sources [46,47] were combined to produce the values used in the crack growth equation. In addition, the initial and final crack lengths are determined independently of the crack growth program. The initial crack length is l_m as justified in section 3.5. The final crack length is determined from the type of failure. If the failure is brittle, the crack length that produces a K equal to the fracture toughness K_{IC} is used as the final crack length. If the failure is ductile, the crack length that produces gross yielding over the net section is used as the final crack length. The computational details are given in Appendix A.

After all the input is determined, the crack growth analysis is performed over the same range of maximum loads as the simplified local strain analysis program, UPLO. The lives calculated for the crack growth are subsequently added to the average of the bounds on crack initiation life, and a prediction for the total specimen life is produced (figure 19b.) This program concludes the life prediction for the specimen. All that remains is to perform the load history reconstructions.

4.4 Rain–Flow Reconstructions

Rain–flow reconstructions are born from the rain–flow cycle count matrix by taking cycles in the matrix and placing them in valid locations in the history under construction. Since the small cycles must reside within the large cycles, the reconstruction procedure begins by introducing the largest cycle and follows by placing cycles in the history in order of declining amplitude. This procedure continues until all the cycles are placed within the history, hence forming a history with an identical rain–flow count.

As the reconstruction process reaches small cycles, the number of valid positions within the history for placement of the small cycles increases. Many choices are available for distribution of

these cycles, as seen in section 3.6. They could be placed deterministically, such as at the beginning of the history, or they could be placed randomly throughout the history. The programs used for the reconstructions in this study take the second option, random distribution, with the additional feature of limits being placed on the distribution of multiple cycles contained in the same location in the rain-flow matrix.

The rain-flow reconstructions are initially performed via the program RECON2 [45]. This program produces a wide array of histories based on two values in the data set that determine multiple cycle distributions, NP and NOC, and also on a seed number for the IMSL random number generation subroutine called by the program. However, the scope of the regenerations is incomplete due to limitations on the relative values of NP and NOC. NP determines the minimum number of cycles in a given rain-flow matrix position necessary for placement in more than one location in the regenerated history. If a value is less than NP, all the cycles are placed in one location in the history. When a value in the the rain-flow matrix is greater than NP, the cycles are divided amongst NOC locations. However, since the FORTRAN statement that divides the number of cycles by NOC performs integer division, NOC must be less than or equal to every value that is greater than NP. In other words, NOC must be less than NP plus one.

This relationship of NP to NOC limits the potential scrambling of cycles. If NOC is large, which effectively divides cycle counts into many positions, then NP must also be large. This restriction prevents the small and medium counts from being divided. If NOC is small, then many more positions are divided. But the large cycle counts are divided into only a few locations. This problem is created by the lack of variability of NOC. Therefore, the most random possible reconstructions are not possible using RECON2. This shortcoming requires a change to the program.

A new program, JRECON, shown in Appendix B, contains changes made to RECON2 that enable a variable NOC. If NP is input as one, then NOC is set equal to the cycle count of each position. Therefore, every position containing a count larger than one is divided into individual cycles, regardless of the count. Dividing the cycle counts in this manner creates an optimum randomness for reconstructions from a given rain-flow matrix.

In addition to the original history (figure 2), two reconstructions created from the program RECON2 and two reconstructions from JRECON are used for the experimental testing. The reconstructions created by RECON2 used in testing are shown in figure 20. The first reconstruction, Reconstruction 1 (figure 20a), is produced from all the cycles for each position in the rain-flow matrix being placed in one location by setting NP equal to a very large number. Reconstruction 2 (figure 20b) is formed by dividing only large blocks (NP=8) into a moderate number of locations (NOC=4). Figure 21 shows the two reconstructions created from JRECON. Reconstruction 3 (figure 21a) has all cycle counts greater than two (NP=2) divided into three locations (NOC=3). Finally, Reconstruction 4 (figure 21b) uses the variable NOC feature (NP=1). For the modified maneuver history rain-flow matrix, Reconstruction 4 is the most randomized history possible.

The reconstructions of the various histories conclude the analytical modeling of the modified maneuver history applied to the notched specimen. Now, experimental testing may begin to verify the validity of the modeling as well as compare the relative severity of different loading histories characterized by the same rain-flow cycle count.

5.0 Experimental Methods

The experimental testing for the study is conducted in two phases. First, the material properties for the 4340 steel, namely the strain life equation constants and the cyclic stress strain constants, are determined. Second, variations on the rain-flow loading spectra, including the original filtered maneuver history and four variations of rain-flow reconstructions are tested on notched specimens at different load levels comparable to those used in the life predictions.

5.1 Determining Material Constants

5.1.1 Test Parameters

The constants for the cyclic stress-strain curve and the strain-life curve are determined from the same set of experiments. The specimen shown in figure 22 is used in a series of constant strain amplitude tests. From each test, the number of cycles to failure is counted and is paired with the strain amplitude to yield a data point on the strain-life curve. In addition, the stress amplitude is measured for selected cycles. Since the material cyclically softens, the stress amplitude is not constant during the life of the test. The value chosen for the stress-strain curve is the stress amplitude at the cycle measured closest to one-half the total life. Therefore, the scope of both the strain-life curve and the stress-strain curve is determined by the range of strain amplitudes used in the testing.

The minimum strain amplitude tested is determined by the material. The lowest strain amplitude should be such that the strain is almost entirely elastic, but not so small that the endurance

limit is reached. This minimum results in a life long enough to garner complete information about the strain-life curve without producing an infinite life. Also, the cyclic stress-strain curve will have values in the elastic range, hence yielding a complete curve.

On the other hand, the maximum strain amplitude is determined by specimen geometry as well as material properties. The load associated with the maximum allowable strain amplitude must be below the buckling load of the specimen. The material could withstand higher strain, but the buckling limits the testing. However, the maximum strain amplitude is chosen so as to get as near as reasonably possible to the buckling load in order to obtain as wide a range as possible on the stress-strain and strain-life curves.

5.1.2 Test Control

Testing is performed on a hydraulic MTS Model 810 10 Kip axial load frame. The frame is controlled with an MTS 458.20 Test Controller, an MTS 458.91 Microprofiler signal generator, and an IBM PS-2/30 personal computer. Initially, the constant amplitude strain tests were controlled by Testlink, an MTS board and software installed in the personal computer.

The first three tests performed on this system were satisfactory. But then, the test system began to show erratic behavior. Twice, the software inexplicably shifted the strain amplitude in a random fashion. The two specimens in the machine during this situation could not produce data since the strain amplitude was not constant. Hence the commercial package Testlink was set aside in favor of a simpler system.

The revised system takes advantage of the remote programmability of the microprofiler. The complete command set controlling the output of the microprofiler is entered into the microprofiler via the RS-232 serial communications port in the personal computer (figure 23). A BASIC program, shown in Appendix C, controls the microprofiler for the constant amplitude strain tests. This configuration performs well and is used for the remainder of the tests.

A final problem, with gripping, occurs with the longer tests. The collet being used to grip the specimen mars the surface of the specimen in the grip, that is, fretting occurs. In one test, a small

crack grew, causing the specimen to fail in the grip instead of in the test section. Further instances of grip failures were avoided by using brass shim stock to protect the portion of the specimen in the grip. This technique works, for no more grip failures happened.

5.1.3 Test Data

During each test, the load and strain are measured for selected cycles. The load, measured by a load cell, is divided by the initial cross sectional area of the specimen to determine the stress. The strain is measured by a half-inch extensometer that is calibrated to produce strain as output. Initially, the stress-strain cycles are measured frequently until the material cyclically softens into a stable hysteresis. Then the cycles are recorded at intervals of approximately five percent of the estimated life until the maximum tensile load begins to decrease. Finally, the cycles are again measured frequently until the specimen fails.

A decrease in the tensile load indicates that a crack is beginning to grow in the specimen. This continues until the specimen separates completely. But since the strain-life data from these tests are needed for predicting crack initiation, the total life of the test is not suitable for calculating the strain-life constants. Instead, once the maximum tensile load falls ten percent below the stable cyclic value, failure is declared. This life is used in conjunction with the strain amplitude to determine the strain-life material constants. Finally, the values of the stress and strain at the nearest measured cycle to half the total life are determined. These data are used to calculate the cyclic stress-strain coefficients.

5.2 Notched Specimen Life Under Variable Loading

The second phase of the testing involves determining the life of a notched specimen. The test system for variable amplitude loading is similar to the final configuration of the constant amplitude tests. However, the test control and data collection are much more difficult than the comparable tasks in the constant amplitude strain tests. The procedure is discussed below.

5.2.1 Test Parameters

The variable amplitude load tests are performed on a notched specimen (figure 24) with an elastic stress concentration factor $k_t=3.92$ [48]. The range of loads used for the tests are the same as the values used in the life prediction, namely a maximum load ranging from 17.8 kN to 40 kN. This choice allows direct comparison between the actual lives and the predicted lives. The actual loading sequence depends on the choice of loading history and the maximum load.

The original history is scaled to and tested at six even increments between 17.8 kN and 40 kN. Since the original history is the primary history of interest, comparison with the predicted lives is a necessary step in the verification of the local strain model. In addition, one other history, Reconstruction 2, is also tested at the entire range of scaling loads to give a complete comparison with the original history. Reconstruction 1 is tested at 22.2 kN, 31.1 kN, and 35.6 kN to yield more data at some of the moderate loads. Finally, Reconstruction 3 and Reconstruction 4 are tested at 31.1 kN to obtain a large variety of histories tested at one load. This choice should bear any variations present amongst the different histories.

5.2.2 Test Control

The computer control of the variable amplitude load tests is similar to that of the constant amplitude strain tests. However, the BASIC program controlling the variable tests is more complicated than the constant amplitude control program, yet load control is easier to operate than strain control. So more effort is spent programming the microprofiler.

The first problem with the microprofiler is related to its 47 kbyte of RAM [49]. The signal output of the microprofiler consists of the test history reversals (peaks and valleys) being connected with haversine segments. Each haversine segment uses 126 bytes in the microprofiler memory. Since each history contains 1020 points, the memory cannot accommodate all the reversals at one time. Hence, the PC program must send the data in blocks of 255. While the microprofiler is executing these values, the PC monitors the count of executed cycles. Before the microprofiler

exhausts all the reversal values in memory, the PC loads the next block of 255 into its memory. But caution must be used, for if the microprofiler empties the reversals in memory, it will turn off. Careful timing is required because the rate at which the PC can download information to the microprofiler is about six points per second. So the variable amplitude tests are conducted at five hertz to guarantee that the RS-232 communications can keep up with the test. Actually, the five hertz rate is suitable because the machine is able to closely match the output with the control signal.

The next problem in programming is rather odd. For some reason that is still unexplained, the microprofiler incrementally offsets the reversal values sent to it. The amount that each value increases is small enough not to affect the test for several blocks of testing. But after a few thousand cycles, the change in mean load is significant enough to stop the test. The only way known to correct this problem is to reset the microprofiler after every completion of the history. This reset complicates the control program, but is necessary to produce a reliable test. With the reset implemented, each test performs very consistently and close to the specified history.

5.2.3 Collecting Data

Three points in the variable amplitude load test are of interest: crack initiation, transition crack length, and specimen failure. The first two points relate to crack initiation. Initiation may be defined as the cycle at which the crack reaches length l_m , as suggested in section 3.5. Or it may be defined as the cycle at which the first crack appears. Experimental detection of these two points in specimen life is very subjective due to the difficulty in measuring small cracks.

The specimen is watched with a Bausch and Lomb stereoscopic microscope at 80X. Several factors inhibit exact crack detection. The specimen is illuminated in the notch with a light source, and the microscope is angled to look inside the notch. Hence, the entire notch cannot be focused simultaneously due to the large depth of field. The initial cracks form in so many different ways, and difficulty arises in spotting the cracks. Since each side of the specimen is watched, time is spent moving the microscope around. Finally, small anomalies appear in the notch that sometimes result in

a crack, and sometimes not. When the first surface flaw grows into a reasonable crack, initiation is declared. However, this point is not definitively defined.

The measurement subjectivity is improved somewhat when l_m is determined. After the crack begins to grow, it propagates through the thickness. Finally it reaches the edge, and the length may be measured. When the length reaches l_m , the second measurement of life is made. But even this measurement contains error. First, the crack sometimes reaches the far side of the notch first, where no measurements of the length may be made. This restricts the transition crack measurement to the front of the specimen. Second, since the crack usually initiates in the center of the notch, growth is expected normal to the notch as well as across the notch. But since this growth is in the middle of the specimen, no measurement of crack length can be made. Nonetheless, the flight at which the crack length reaches the transition length, l_m , on the front side of the specimen is recorded and reported as the life to l_m .

The number of flights to failure is the final measurement made on the notch specimen test. This measurement contains no subjectivity, for the failure of the specimen determines the data point. Therefore, the number of flights to failure is the easiest to determine and the most reliable of the three measurements.

6.0 Results

The data from the experimental testing are presented in the order of testing. The first part is the presentation of the constant amplitude data and the resulting strain-life and cyclic stress-strain constants derived from the data. The second part is the presentation of the lives measured for crack initiation and failure.

6.1 Constant Amplitude Strain Tests

6.1.1 Cyclic Stress-Strain Equation

Table 2 shows the constant amplitude strain levels and the corresponding stress amplitudes and lives. The data for the stress and strain amplitudes are fit to the cyclic stress-strain equation, equation 3.3. The elastic modulus, E , is found by averaging the value measured from each test. Then the elastic strain amplitudes are determined from the stress amplitudes and subtracted from the total strain amplitudes to yield the plastic strain amplitudes. These data are plotted on log-log plot and a least squares fit is done. The resulting constants for the cyclic stress-strain curve are shown in Table 1. Figure 25 summarizes the results of the cyclic stress-strain curve by showing the actual data points and the fitted curve given by the above material constants.

6.1.2 Strain-Life Equation

The data for the strain amplitude as a function of the life are fitted to the strain-life equation, equation 3.6. Since the strain-life equation produces two straight lines on a log-log plot, two least square fits must be done. One line is the plastic strain amplitude versus life, and the other is the elastic strain amplitude versus life. Each line determines the respective constants in the strain-life curve. The values determined from the data are shown in Table 1.

Figure 26 summarizes all the data for the strain-life curve. Each measured data point consists of the total strain amplitude as a function of life. Since the total strain amplitude is the sum of elastic and plastic portions, both of these values are also shown. For long lives, the plastic portion of the strain amplitude is negligible and is not shown on the plot. Notice that the elastic and plastic strain amplitudes for each life combine to give the total strain amplitude. The linear fits to these lines are also shown for completeness.

The two equations just described complete the determination of the material constants. These constants are then applied to the computer modeling described in chapter 4 to predict the life of the notched specimen. The results of the testing of the different histories on the notched specimen may now be compared to the predicted values for life.

6.2 Notched Specimen Lives Under Variable Amplitude Loading

Table 3 gives the lives for crack initiation, transition crack length, and specimen failure as a function of maximum load in the loading sequence. The data from this table are then plotted in three separate plots.

Figure 27 shows the flights to crack initiation for all of the histories as a function of the maximum applied notch stress, $k_t S_{max}$. Also on this plot are the predicted bounds for crack initiation life from the program UPLO described in chapter 4. Similarly, figure 28 shows the flights to transitional crack length, l_m , for all the histories. Again, the predicted crack initiation bounds are

plotted with the data. Comparison with the predicted bounds shows the relative magnitudes of the effects of the two definitions of crack initiation.

Finally, figure 29 shows the flights to specimen failure as a function of $k_t S_{\max}$. Here the predicted total life is plotted for comparison. The total life is calculated by adding the results of the crack initiation program, UPLO, with the results from the crack growth program, CRACK. The three points of the specimen life may now be compared to verify the life prediction and test equivalency of the different histories.

7.0 Discussion

The data from the experimental testing are discussed in the order of testing. An evaluation of the material constants is presented first, followed by a discussion of the variable amplitude load tests.

7.1 Material Constants

As seen in the results section, only nine valid tests are used to determine the cyclic stress–strain and strain–life equations. Nine is a marginal number of specimens to calculate constants involving fatigue. Fortunately, the scatter is not bad, and the data have a good correlation with the curves. Normally, more tests are performed in the region where the plastic and elastic strain amplitudes are of similar magnitudes. However, more specimens could not be obtained, so the material constants are computed with the data available.

The worst part of the specimen shortage in the strain–life testing is the lack of any data for the overstrain effect. This effect may shorten the predicted fatigue life, especially in the lower level testing where the strain amplitudes are more elastic in nature. The 4340 steel is probably affected more in the endurance limit value [15] and less in the character of the strain–life equation. Nevertheless, the final result is that the life predictions may be too high in the simplified analysis for the lower load levels.

The constants in the cyclic stress–strain equation agree very well with other materials with similar monotonic characteristics. This agreement suggests that the constants accurately describe

the stress-strain behavior of the 4340 steel under cyclic conditions. The constants for the strain-life curve, however, appear to be different than the values for other similar materials. Two reasons for the difference are declaration of failure during the test, and the sensitivity of the constants to curve fitting. Since failure was declared in the constant amplitude strain test when the load reduction was ten percent, these values may differ from other tests. The ASTM standard is not concise on this point. Potentially, the specimen lives could be longer if the failure condition is relaxed to a lower tensile load or complete failure.

Since the two coefficient terms in the strain-life curve, σ'_f and ϵ'_f , are determined by y-intercepts on a log scale, their absolute magnitudes are sensitive to small changes in the slope of the line. Comparison of the strain-life curve derived from these constants to other curves of similar materials shows little difference in the actual character of the curve. The two above observations lead to the conclusion that the material constants derived from the constant amplitude tests are accurate for the 4340 steel with a 1500 MPa ultimate strength since the data form a well defined curve.

7.2 Specimen Lives Under Variable Amplitude Loading

The results of the different histories on specimen life are noteworthy. The two crack initiation conditions show markedly similar character when compared to the predicted life to crack initiation. Since the transition crack length is obviously reached after a crack begins, using the cycles to crack initiation is more conservative in predicting the specimen life. However, for the higher load levels, the predicted bounds are conservative for both definitions of crack initiation. This probably results from Neuber's rule overestimating the notch strain at higher stress levels [28]. Anyway, the consistency of the lives for the different histories is good.

For the longer tests, the lives of all the histories fall below the predicted bounds. A large contribution to this effect is the strain-life equation used in the life prediction. Since the overstrain effect is not incorporated into the strain-life equation, the damage done by the lower level cycles may

be underestimated. This effect would be most apparent in the experiments tested over the lower load values. In addition, a strain–life curve based on a larger data set might also improve the correlation.

Finally, regardless of the life predictions, the similarity of the different histories is noted. Since the measurement of crack initiation is the the most subjective, the scatter in the data for this point is expectedly moderate. The scatter in the data of the transition crack length is also fair. But the histories that produce the shortest life to crack initiation do not always produce the shortest life to the transition length. This observation suggests inconsistency in the measurement system. Finally, the data for flights to failure contain no subjectivity and produce the least scatter. The conclusion is that the reconstructed histories yield testing lives equivalent to the original history.

8.0 Conclusions

The following conclusions are drawn from the experimental tests:

- The rain-flow reconstructed histories produce experimental lives comparable to the lives of the original filtered helicopter load spectrum for both crack initiation and failure.
- The simplified analysis derivation of the local strain approach reasonably predicts the fatigue crack initiation of a notched component subject to variable amplitude loading.
- The life predictions may be improved from further investigation of the overstrain effect and a broader scope of validation tests.

9.0 Summary and Recommendations

The testing of the the filtered maneuver history on the notched test specimens further supports the local strain analysis with rain-flow cycle counting as a viable tool for fatigue life prediction. The completed tests further the confidence in rain-flow reconstructed histories as substitutes for the original history in the interest of saving time and data storage. However, further work is recommended.

It would be desirable to determine the overstrain effect on the strain-life curve for 4340 steel with an ultimate tensile strength of 1500 MPa. Including this effect in the life prediction could produce lives at lower stress levels more comparable with the experimental values. However, the extent of this effect is unknown in this application. And since the values are non-conservative, validation of the theory that the overstrain effect is the cause of this problem is important for confidence in the accuracy of the local strain approach.

More tests can be performed to refine the framework in which the rain-flow reconstructions are applicable. The tests performed in this study used a rain-flow matrix of 32 by 32. By changing this size, the effect of the cycle counting resolution on the specimen life could be investigated. In addition, deterministic reconstructions similar to those produced by Perret could be tested at a variety of load levels to further investigate the ordering of cycles. Finally, the testing should be performed on specimens with different stress concentration factors. This would verify that the technique is more universally applicable. At this point, though, the rain-flow regeneration method shows promise for component testing.

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Table 1 Material Properties for 4340 Steel

	<u>Property</u>	<u>Value</u>
	E, GPa	205
	Yield, MPa	1430
	Ultimate, MPa	1500
Cyclic Stress-Strain Equation	A, MPa	2070
	s	.142
Strain-Life Equation	σ'_f , MPa	1680
	b	-.078
	ϵ'_f	.23
	c	-.52
Crack Growth	C	7.9×10^{-11}
	m	3.34
	ΔK_{TH} , MPa \sqrt{m}	5.0
	K_{IC} , MPa \sqrt{m}	170
	γ	.32

Table 2 Constant Amplitude Strain Data for Smooth Specimen

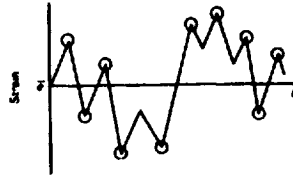
ϵ_a , m/m	σ_a , MPa	N, Cycles
.0026	535	2,910,000
.0036	742	29,200
.0043	815	23,100
.0053	838	6510
.0070	882	3360
.0078	896	2110
.010	965	742
.015	1095	158
.025	1204	63

Table 3 Variable Amplitude Load Data for Notched Specimen

<u>Maximum Load, kN</u>	<u>History</u>	<u>First Visible Crack</u>	<u>Critical Crack Length</u>	<u>Failure</u>
40	Original	5.4	11	35
	Reconstruction 2	5.9	15	33
35.6	Original	9	25	55
	Reconstruction 1	12	17	48
	Reconstruction 2	11	21	48
31.1	Original	21	28	80
	Reconstruction 1	28	41	112
	Reconstruction 2	27	40	89
	Reconstruction 3	16	34	90
	Reconstruction 4	19	39	83
26.7	Original	34	49	124
	Reconstruction 2	36	54	144
22.2	Original	61	141	239
	Reconstruction 1	40	145	235
	Reconstruction 2	65	125	254
17.8	Original	230	575	725
	Reconstruction 2	168	415	633

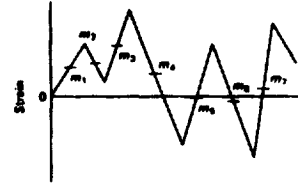
PEAK COUNT

Local Maxima above mean and minima below mean are counted.



RANGE MEAN COUNT

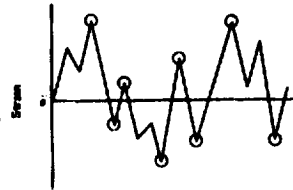
This method adds the mean value to the range count.



MEAN CROSSING

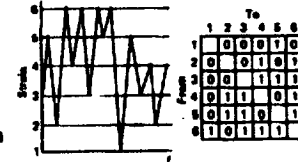
PEAK COUNT

Greatest maxima and minima between level crossings are counted.



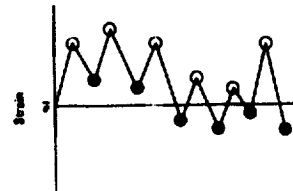
MARKOV MATRIX COUNT

Successive reversals are counted as transitions from one level to another, and stored in a 2-dimension distribution.



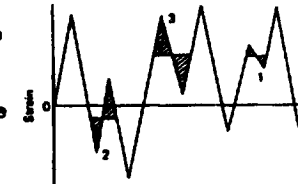
PEAK AND VALLEY COUNT

All reversals counted. Two distributions describe maxima and minima.



RANGE PAIR COUNT

Ranges and means are counted from small to large by continually editing the history. The second figure shows the history after some editing.



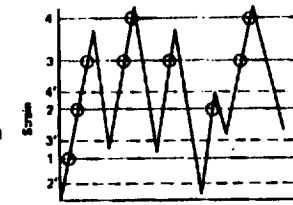
LEVEL CROSSING COUNT

Each level crossing is counted for increasing values above mean and decreasing values below mean.



FATIGUE METER

Same as level crossing except a value other than the mean may be chosen for crossing.



RANGE COUNT

Each range, the difference between successive reversal points, is counted.



RAIN-FLOW CYCLE COUNTING

Method is named after water flowing off multiple roofs. Cycle information is same as range-mean count, except that cycles are extracted sequentially. For some portion of a history shown in first figure, the method counts cycles that are equivalent to closed hysteresis loops.

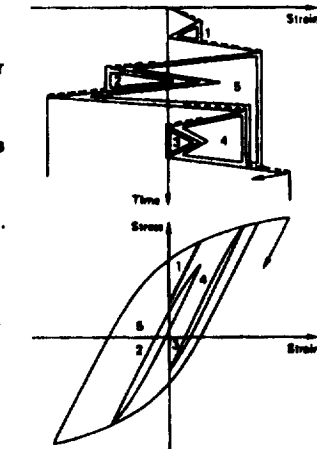


Figure 1 Variable History Characterization. Adapted from [7].

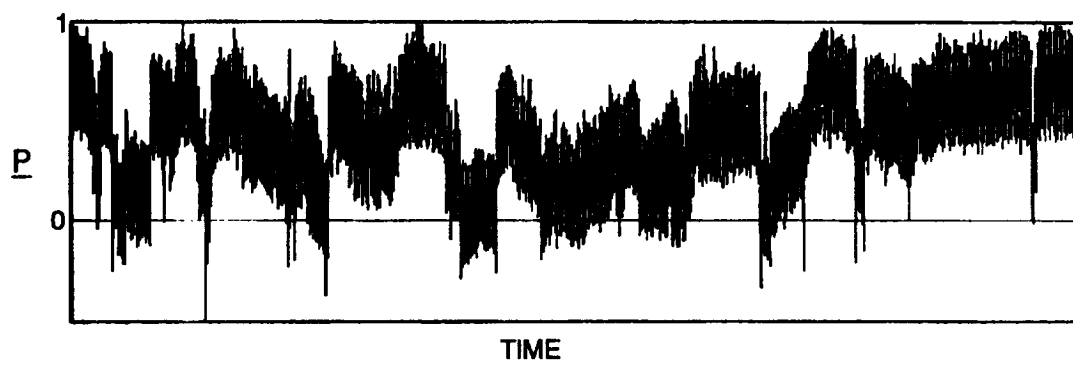


Figure 2 Normalized Filtered Maneuver History for Helicopter Tail Rotor Pitch Beam.

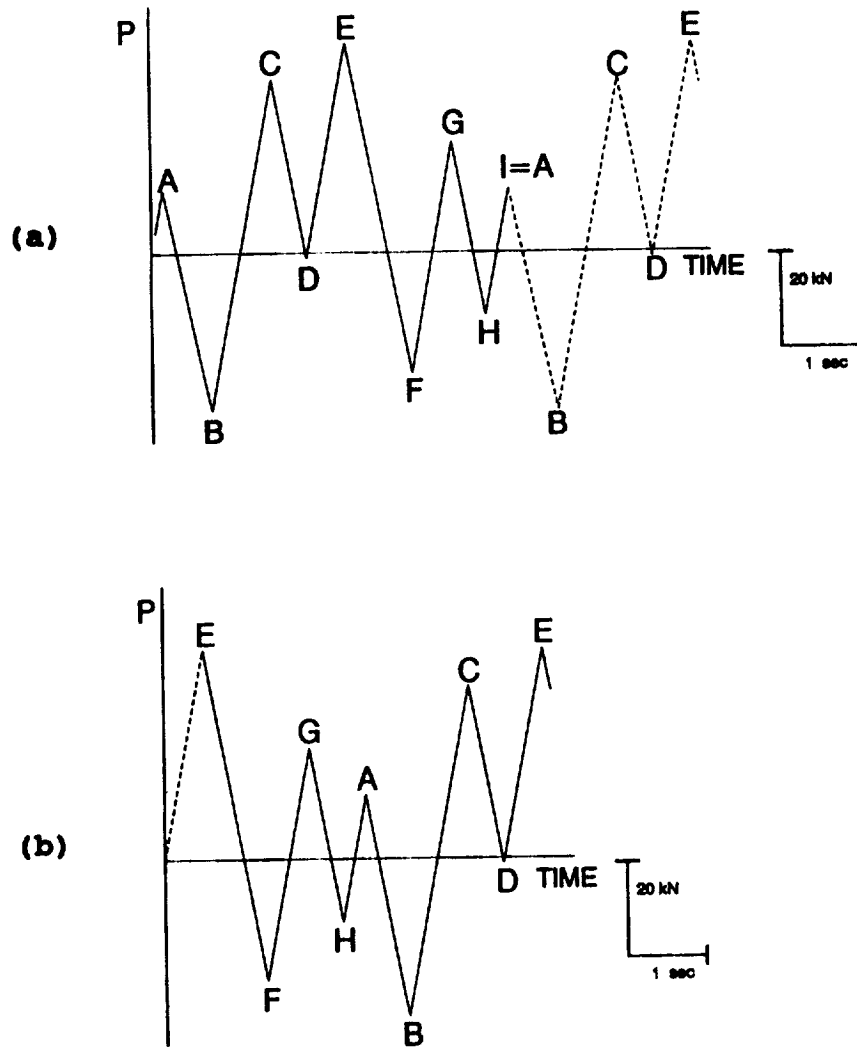


Figure 3 Reordering of Peaks for a Repeating History. The repeating history (a) is changed so that the largest peak occurs first (b). Adapted from [8].

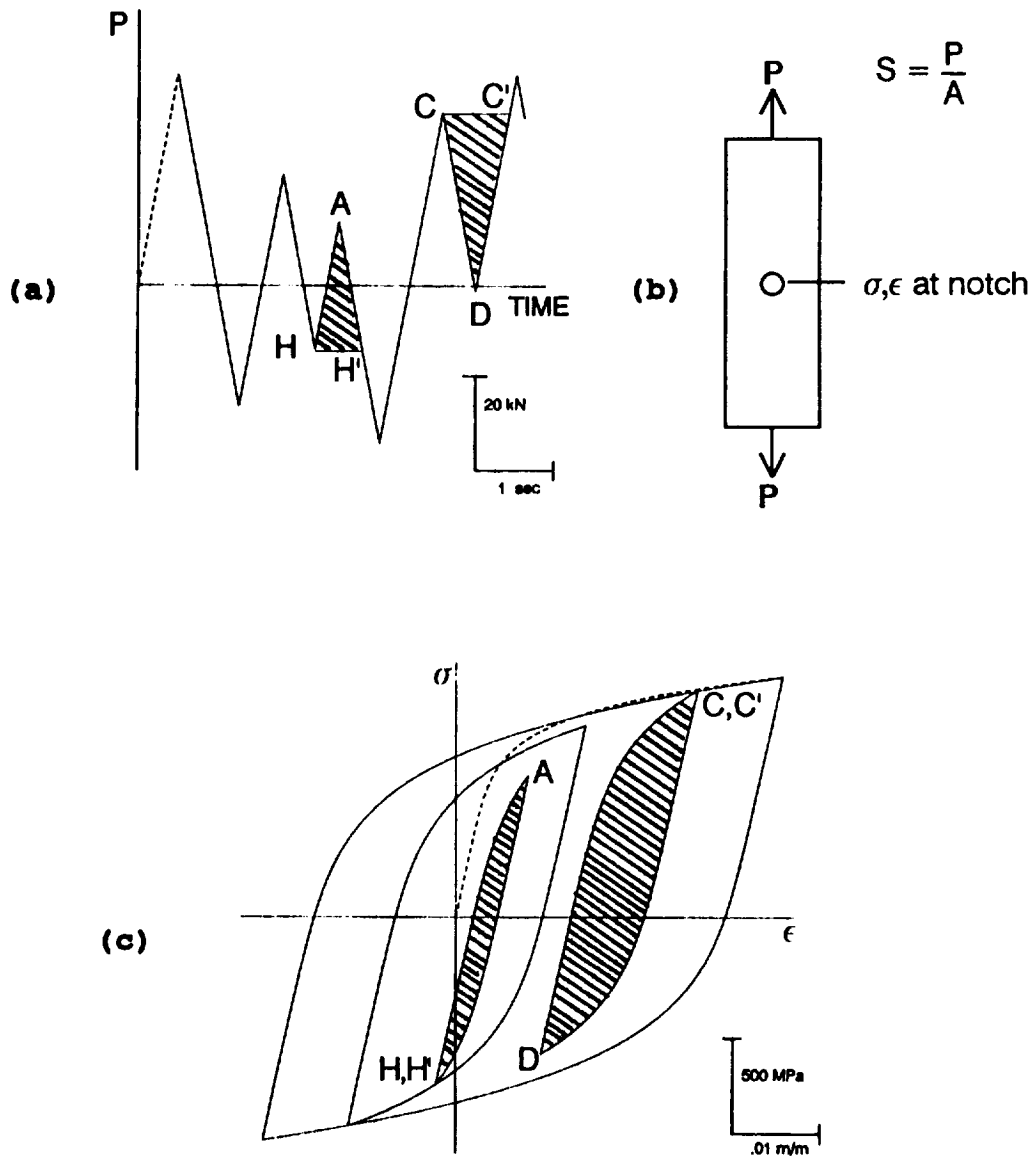


Figure 4 Determination of Rain-Flow Cycles. The variable history (a) applied to a notched specimen (b) produces hysteresis loops (c) for the notch material. All load reversals that produce loops are considered cycles.

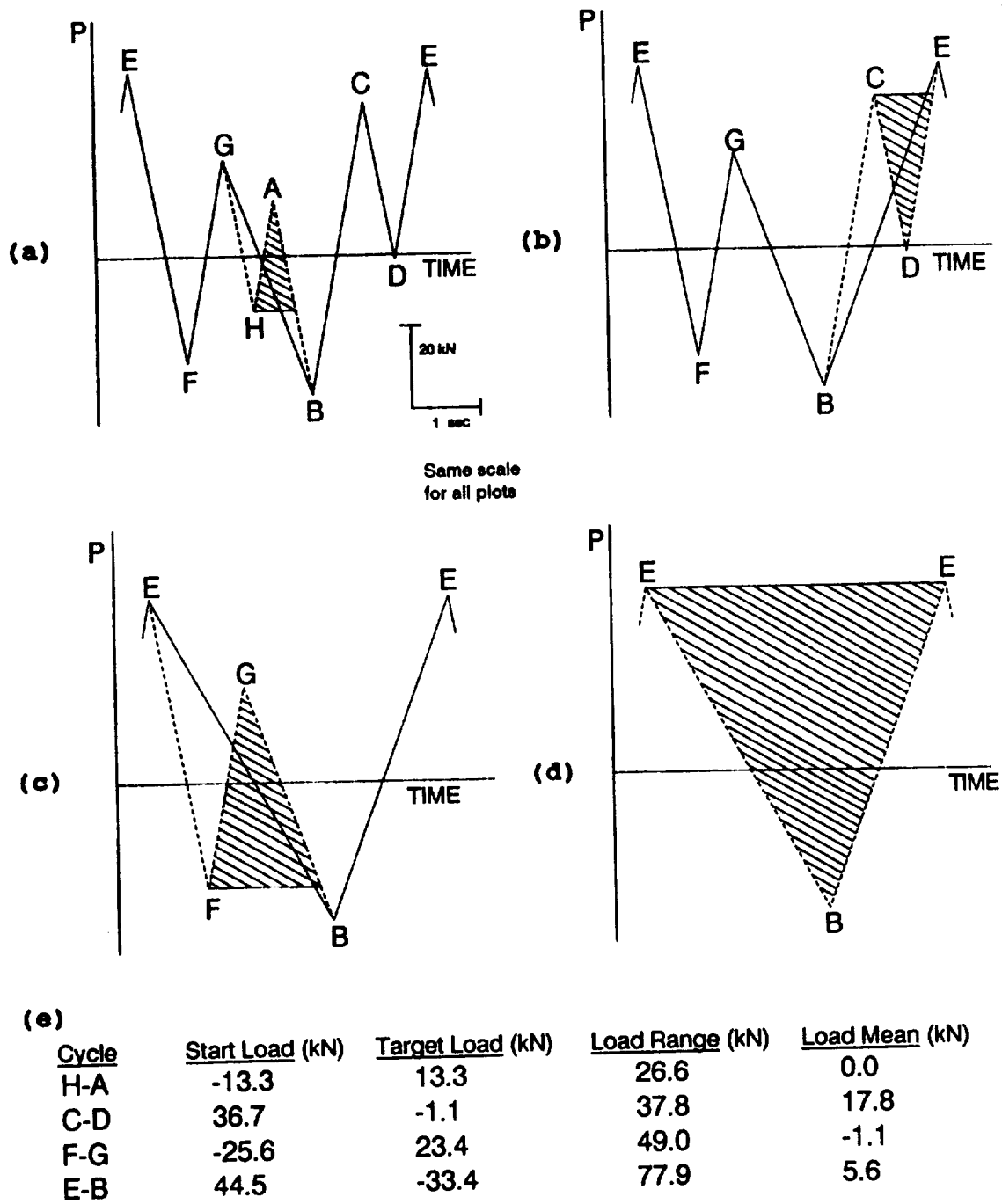


Figure 5 Extraction of Rain-Flow Cycles. Each time a cycle is counted (a-d) the reversals are removed and the cycle value is entered into a table (e). Adapted from [8].

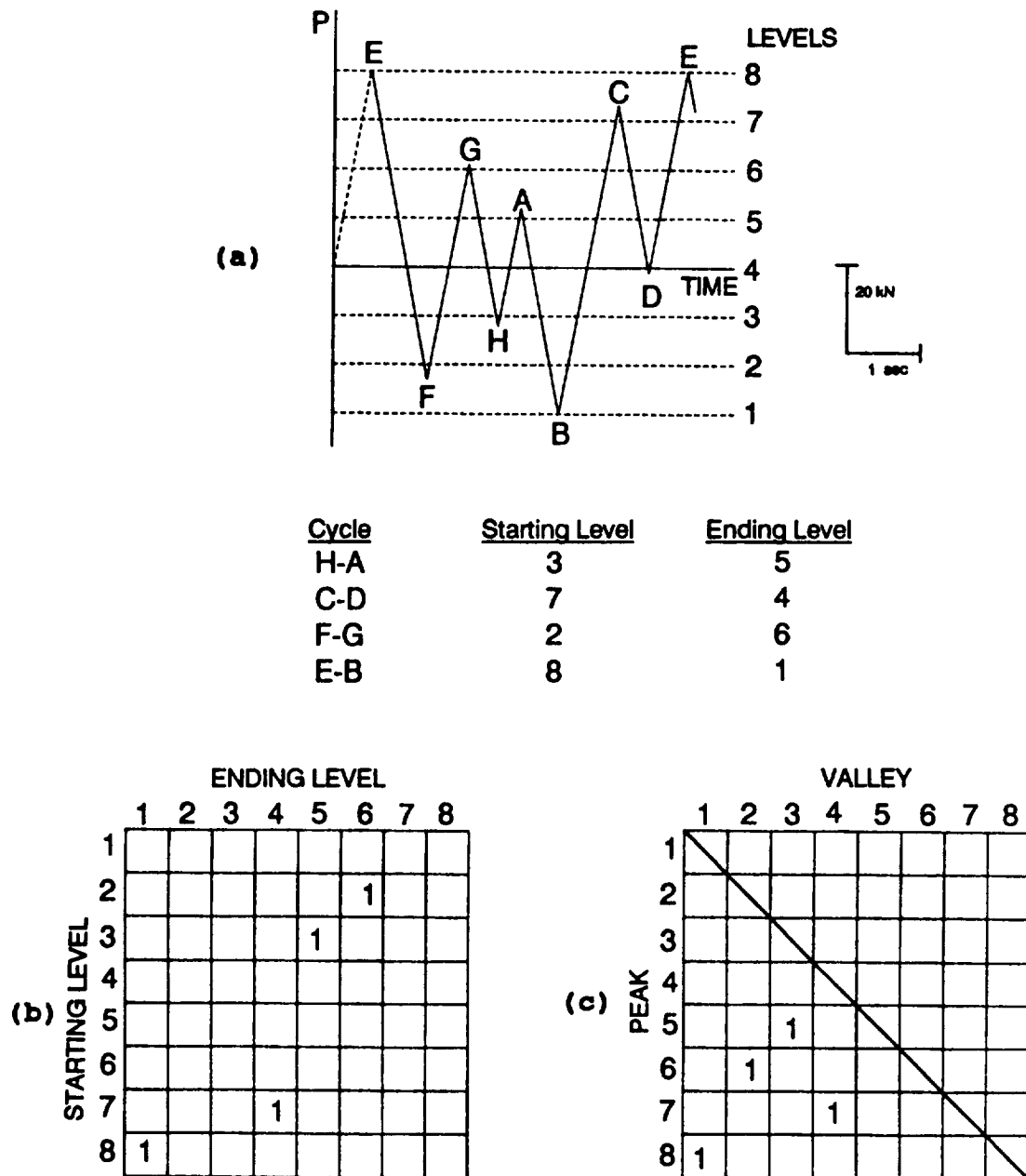


Figure 6 Discretization of Load Levels. Extracted cycles (figure 5) are recorded by level (a) and placed into a rain-flow matrix either containing cycle directions (b) or simply the cycle peak and valley (c). Adapted from [17].

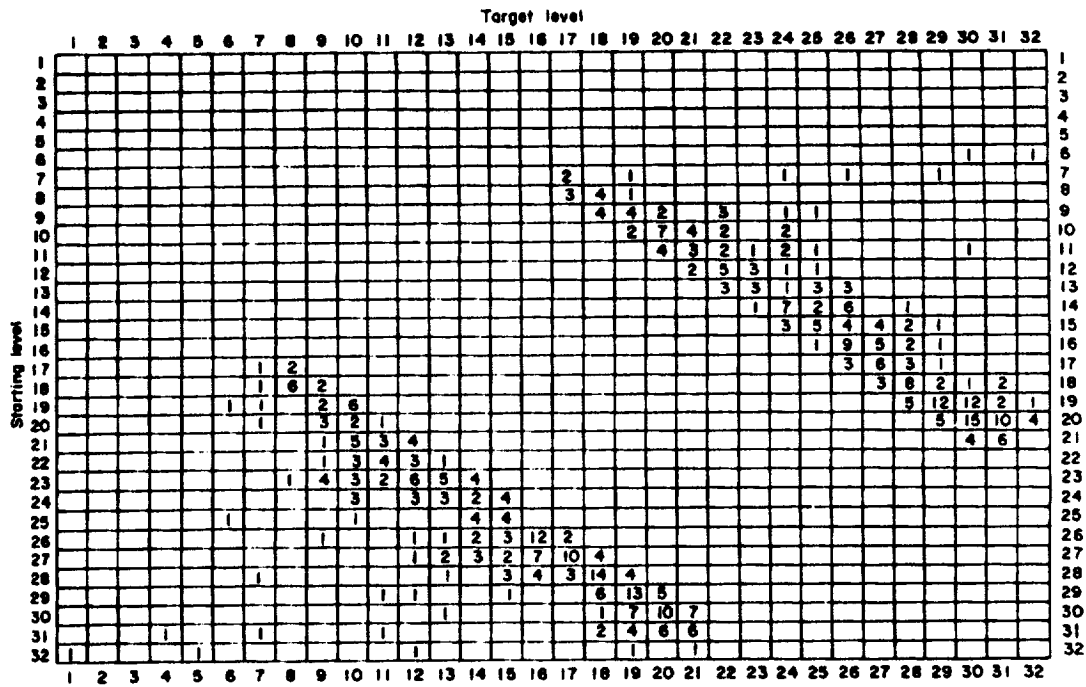


Figure 7 Rain-Flow Matrix for Filtered Maneuver History [17].

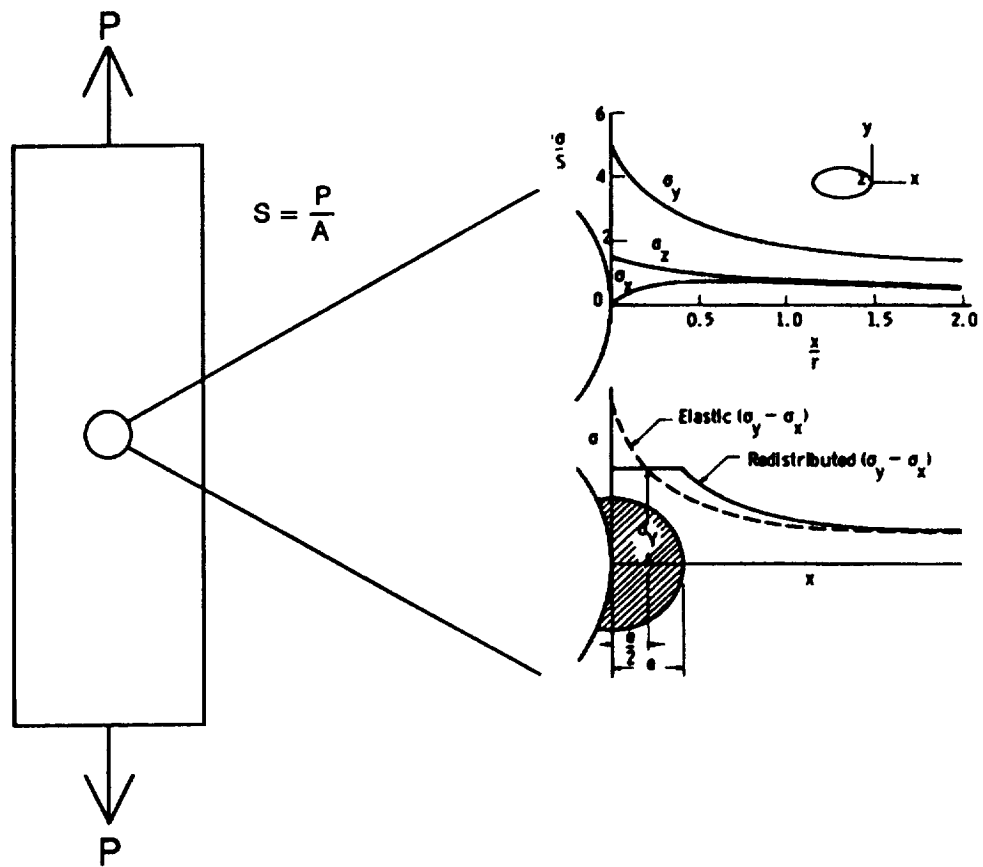


Figure 8 Redistribution of Notch Stress Due to Material Yielding [28].

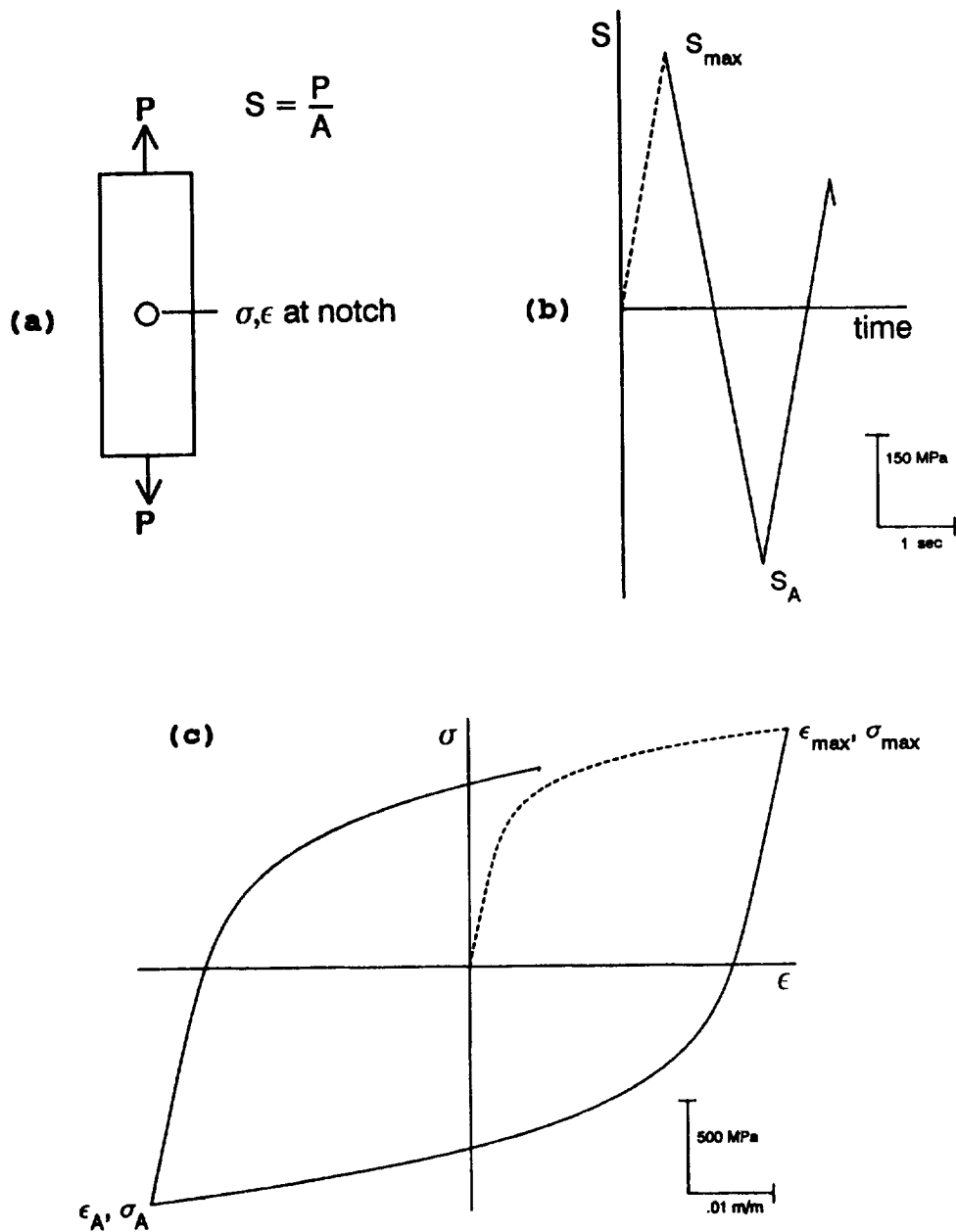


Figure 9 Description of Notch Stress-Strain Response. When the notched specimen (a) is first subjected to the maximum load (b), the equations compute the maximum stress and strain (c). Thereafter, the loop size is doubled, like moving to point A.

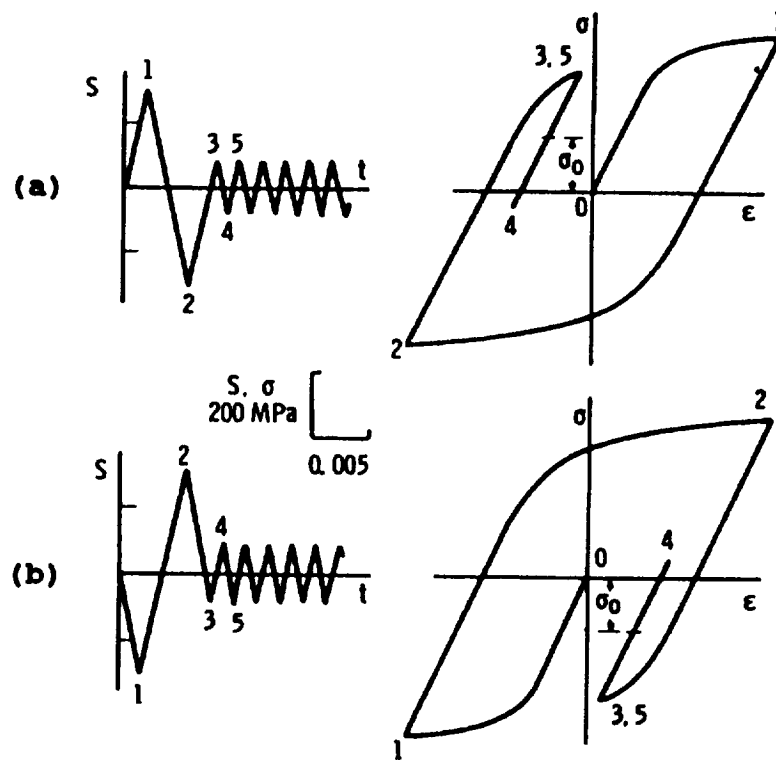


Figure 10 Dependence of Notch Stress on Cycle Order. Note that each nominal stress cycle has zero mean after the third cycle. But the local notch stress is almost completely tensile in one case (a) and completely compressive in the other (b) due to the ordering of S [50].

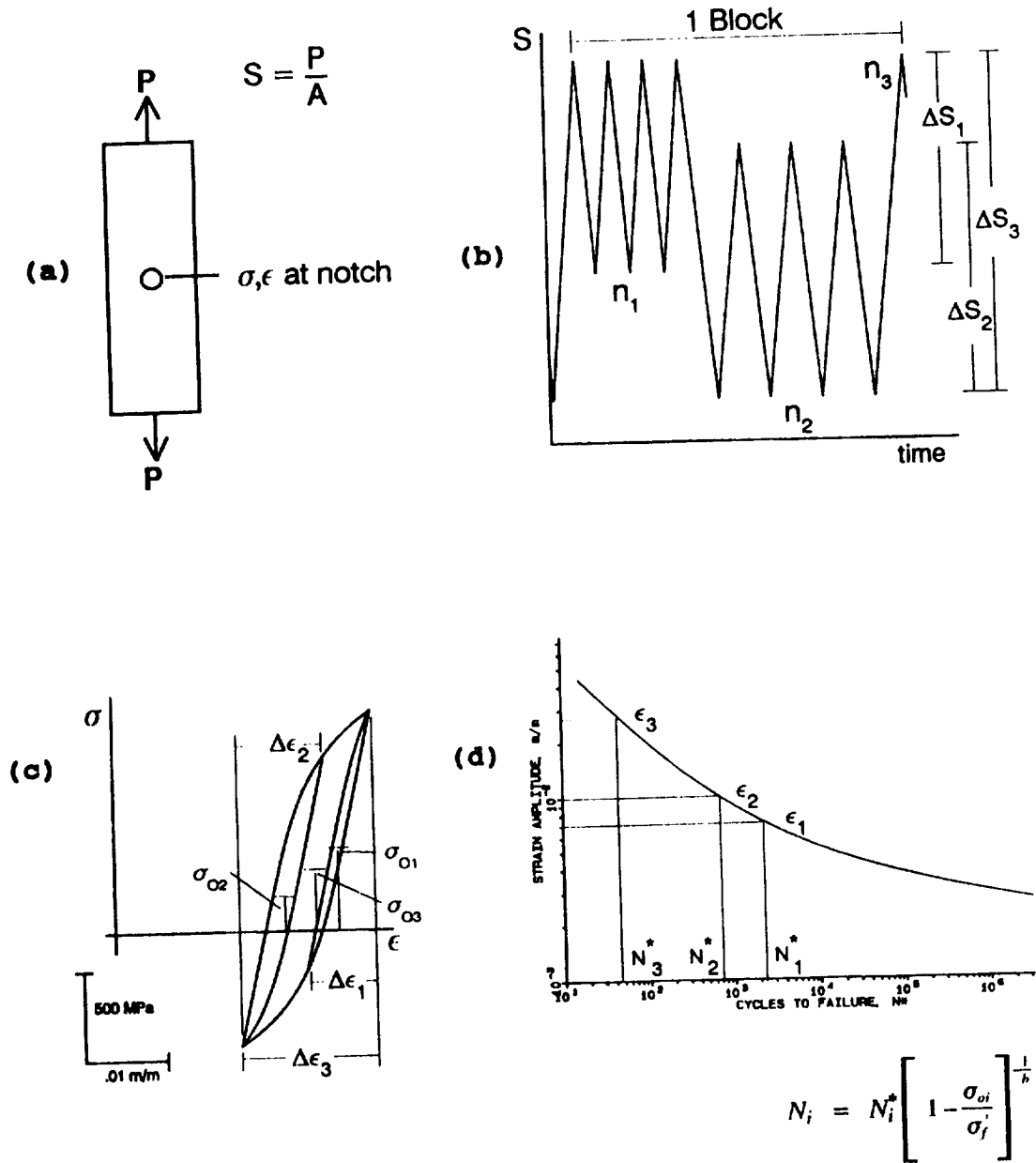


Figure 11 Summary of P-M Rule. A notched specimen (a) subject to a repeating load (b) produces hysteresis loops (c) in the notch material response. Strain ranges are then used to compute life, which is subsequently modified by mean stress (d) for the life fraction summation. Adapted from [50].

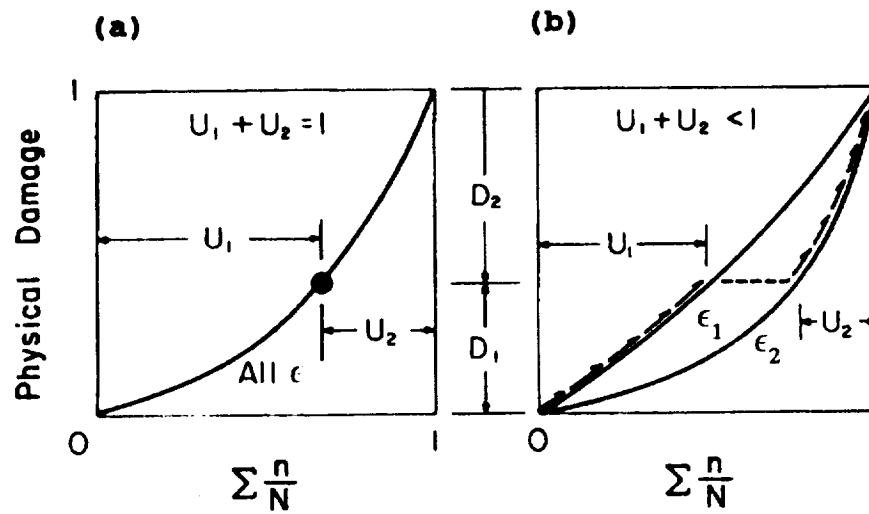


Figure 12 Summation of Life Fractions. A unique relationship between damage and life fraction ensures a summation value of one (a). If damage rate is dependant on amplitude (b), life fraction sums may be less than one for failure [15].

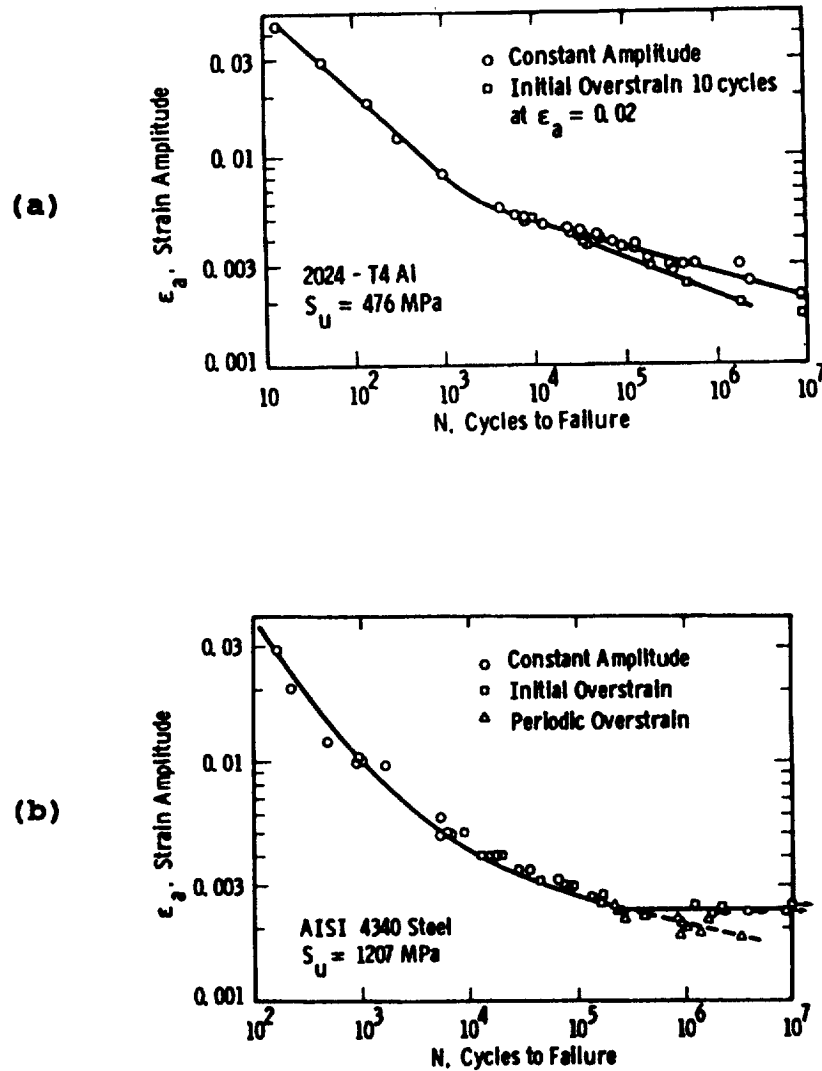


Figure 13 Overstrain Effect. Overstrain during constant amplitude tests may change strain-life equation (a) or eliminate endurance limit (b) [50].

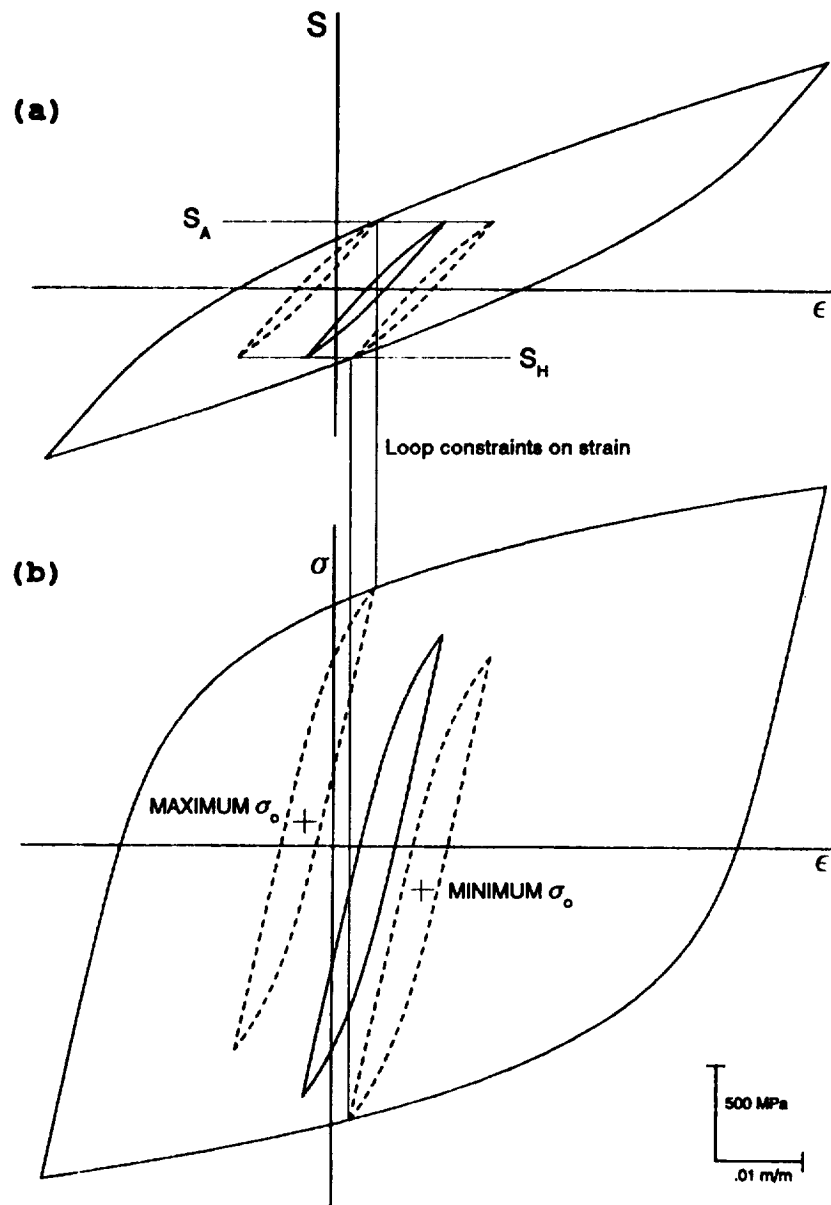


Figure 14 Mean Stress Bounds. For a specified load range, the cycle loop must reside within the largest loop (a). This restriction fixes the strain limits which subsequently restrict the mean stress limits (b). Adapted from [22].

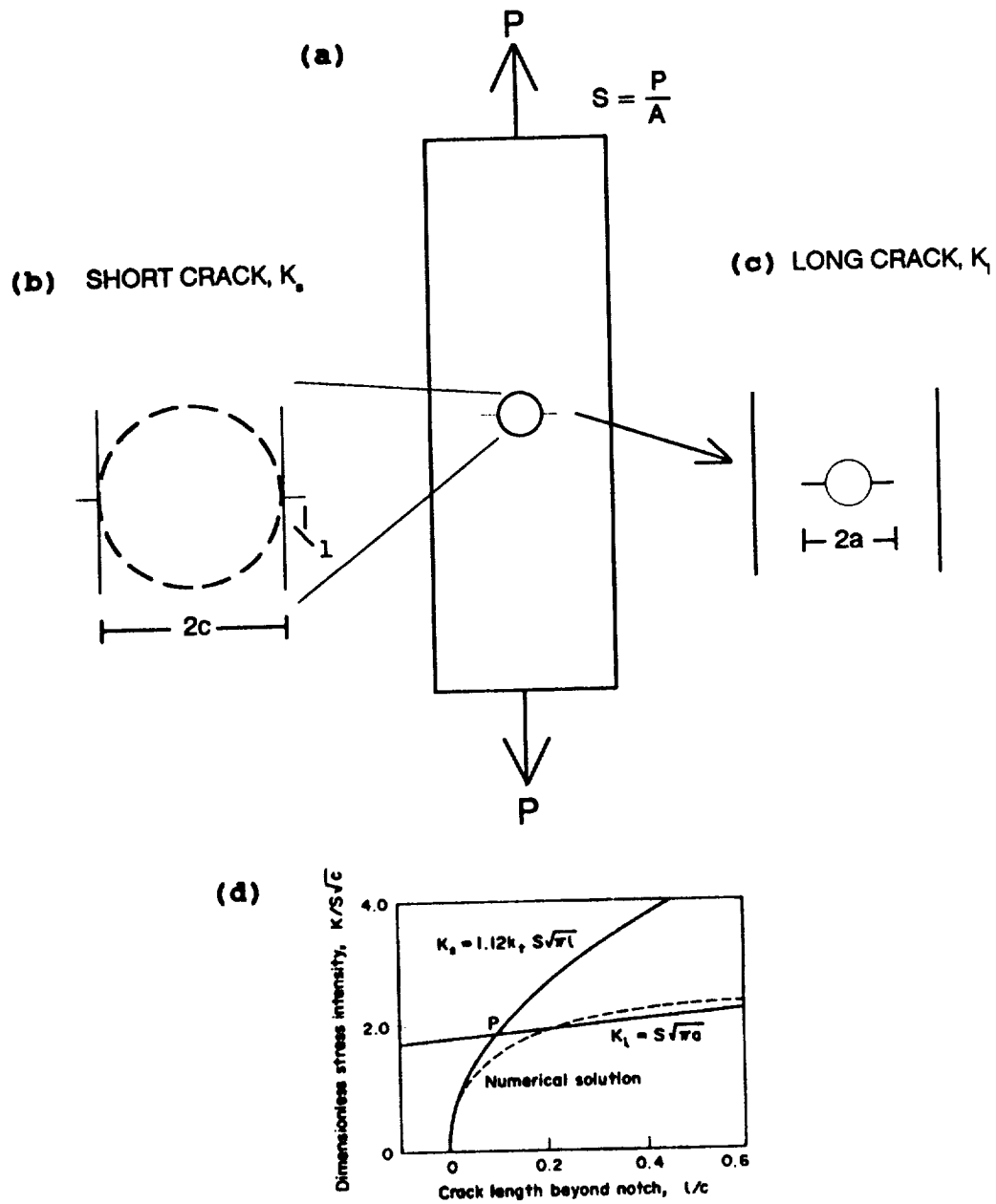


Figure 15 Stress Intensity Transition for a Center Notched, Cracked Specimen. The cracked specimen (a) behaves as an edge-cracked specimen (b) until long crack behavior (c) defines the transition (d). Adapted from [22].

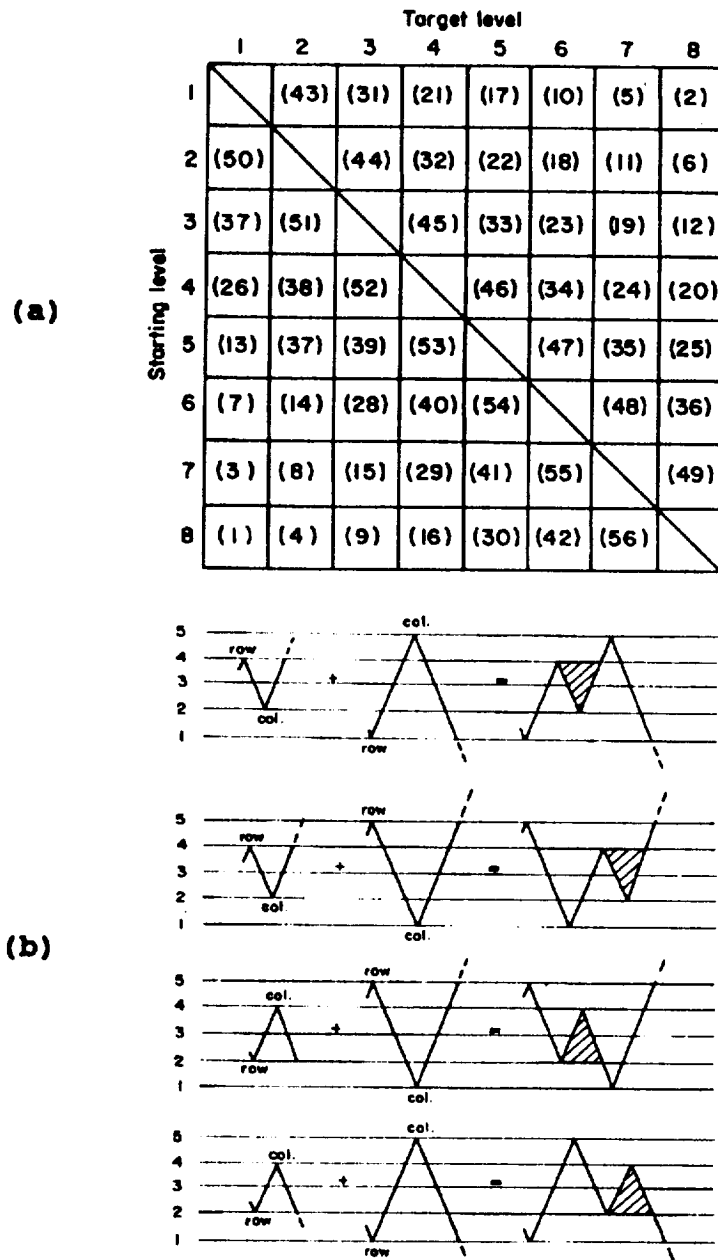


Figure 16 Guidelines for Rain-Flow Reconstruction. The cycles are inserted from largest to smallest (a) governed by the choice of valid insertion sites (b) [17].

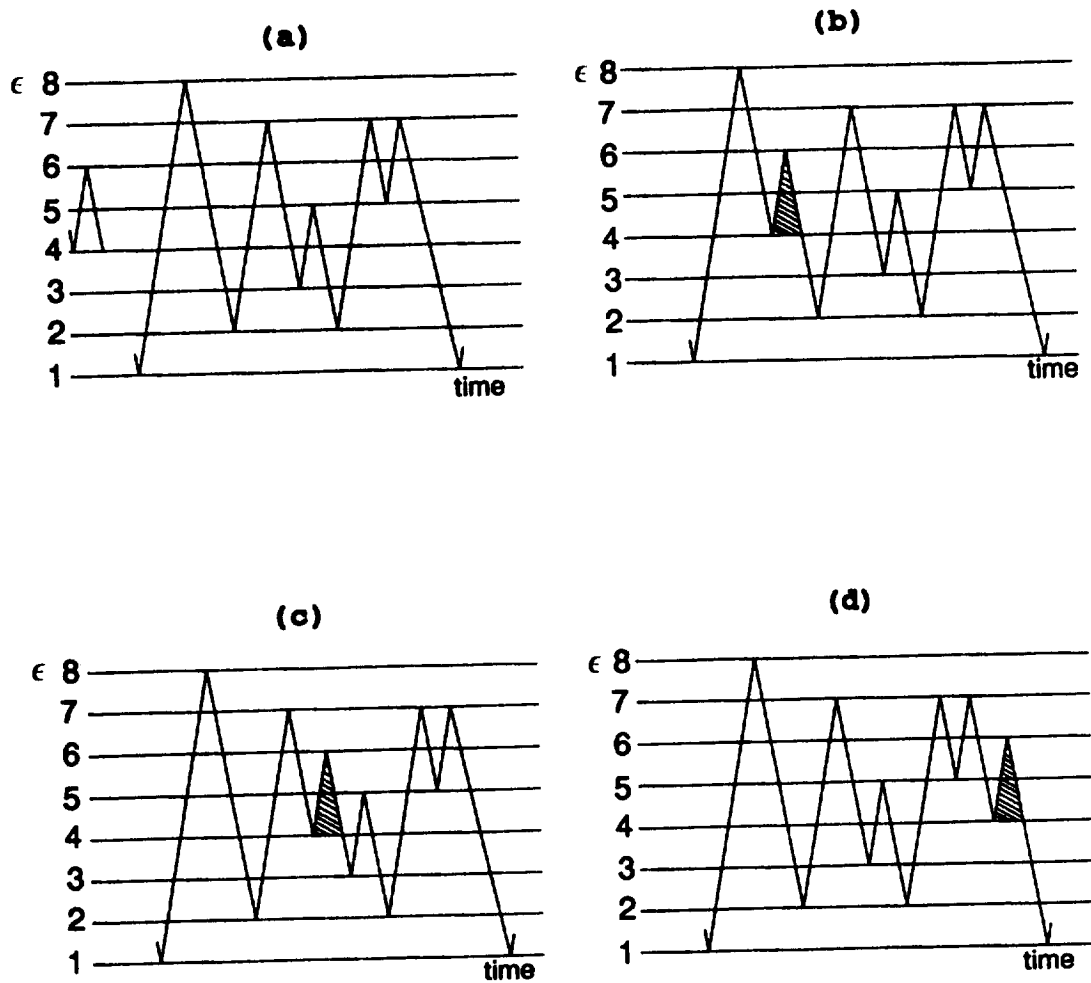


Figure 17 Valid Insertion Sites for an Ascending Cycle. Inserting cycle 4–6 into a short history (a) yields three different histories (b–d). Adapted from [17].

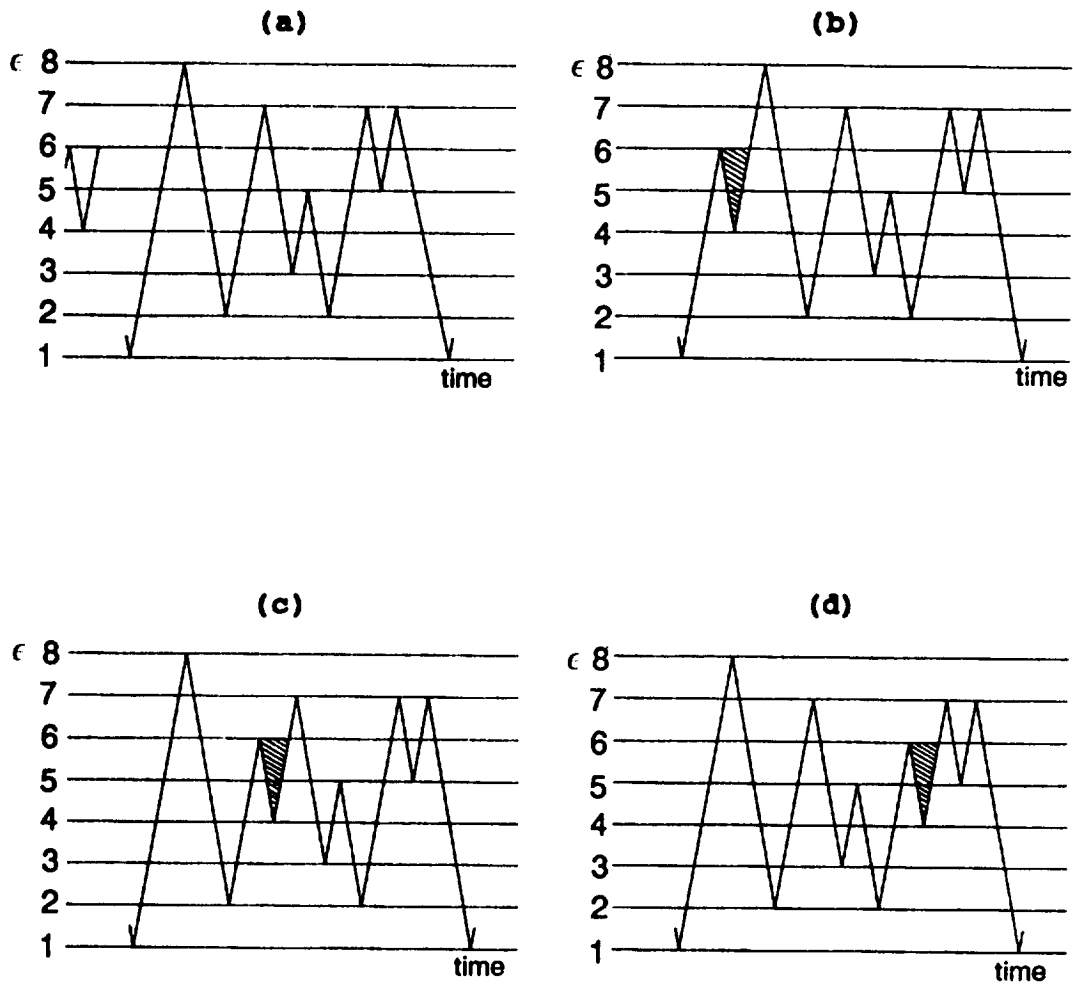


Figure 18 Valid Insertion Sites for a Descending Cycle. Inserting cycle 6–4 into a short history (a) yields three different histories (b–d). Adapted from [17].

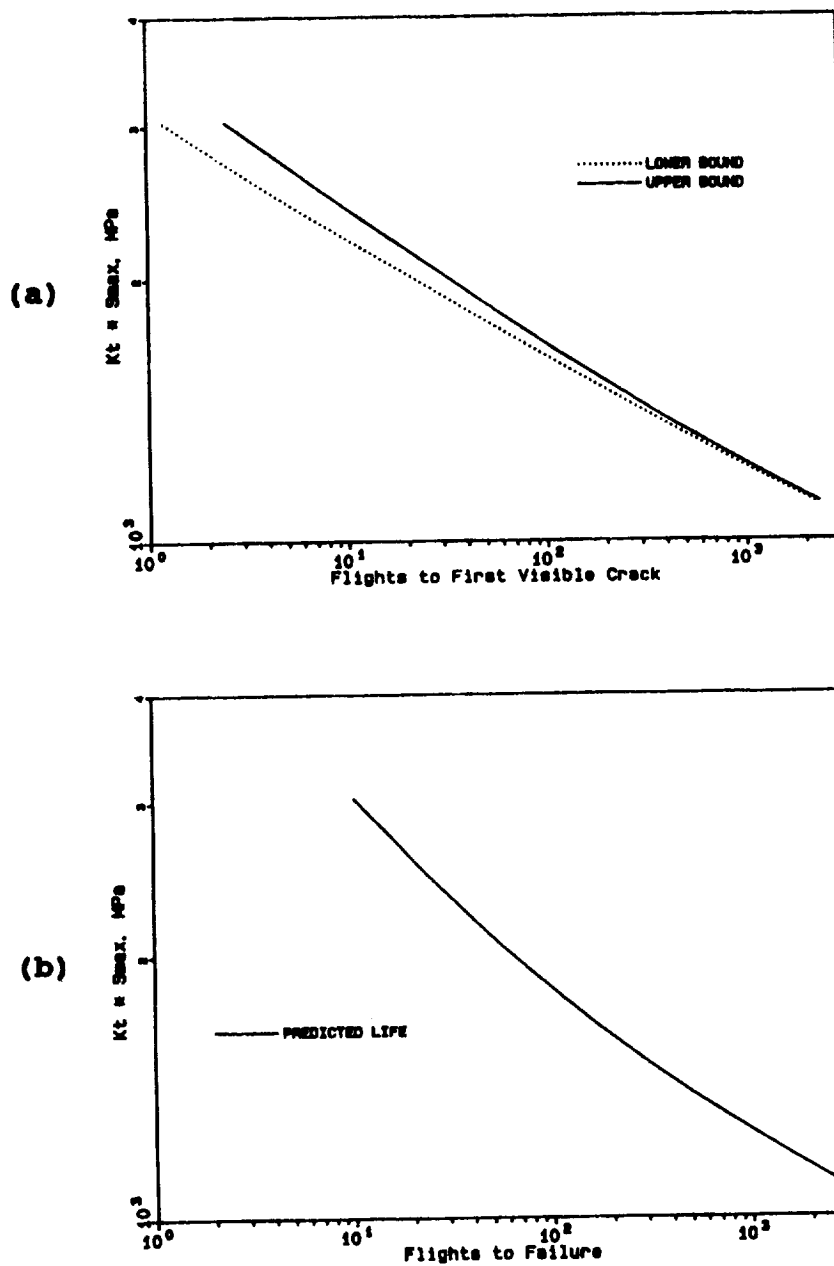


Figure 19 Predicted Life Bounds. Predicted life in flights for crack initiation (a) and total failure (b) for the notched test specimen.

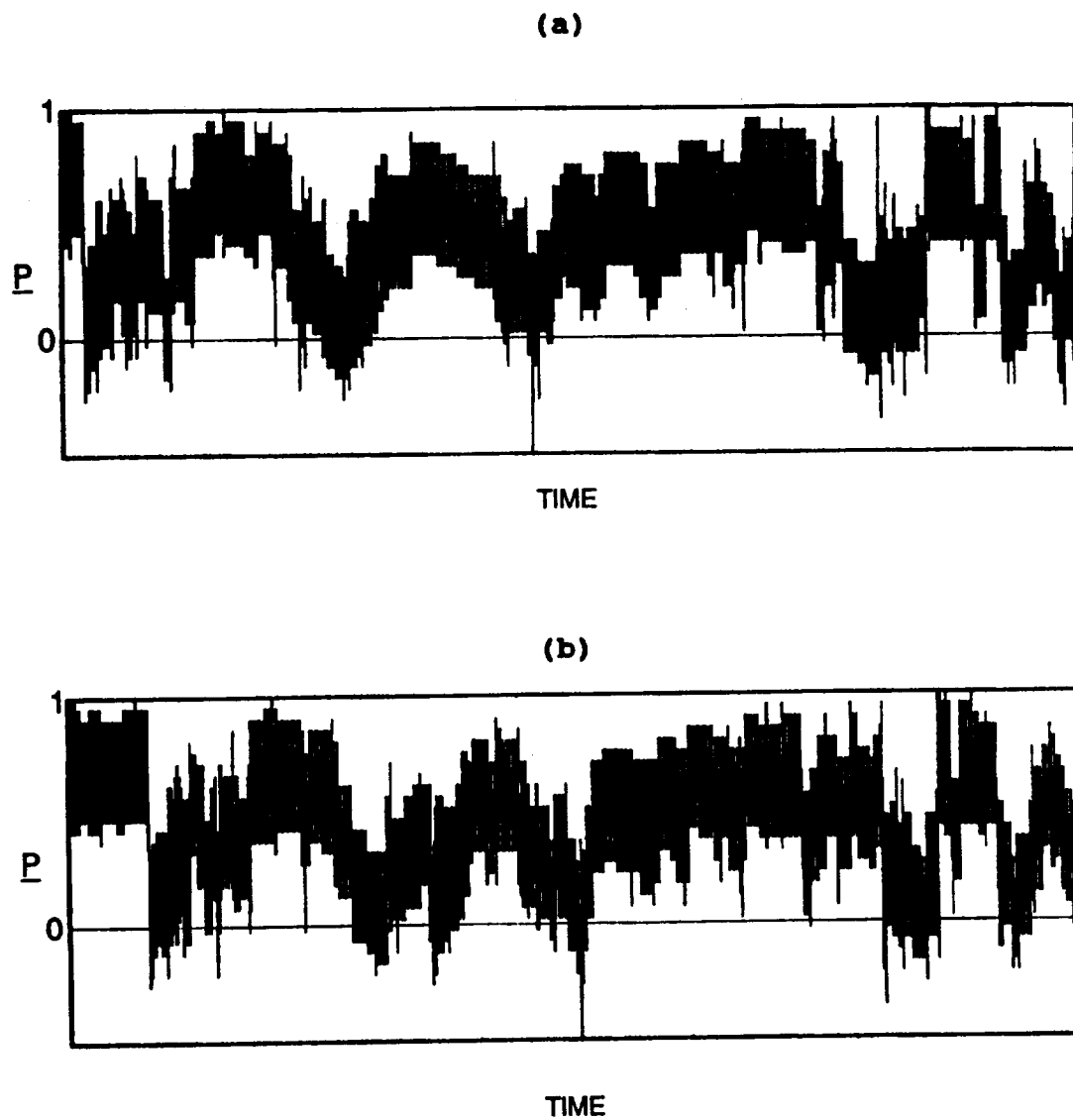


Figure 20 Reconstructions from Program RECON2. Reconstruction 1 (a) and Reconstruction 2 (b) contain only moderate scrambling.

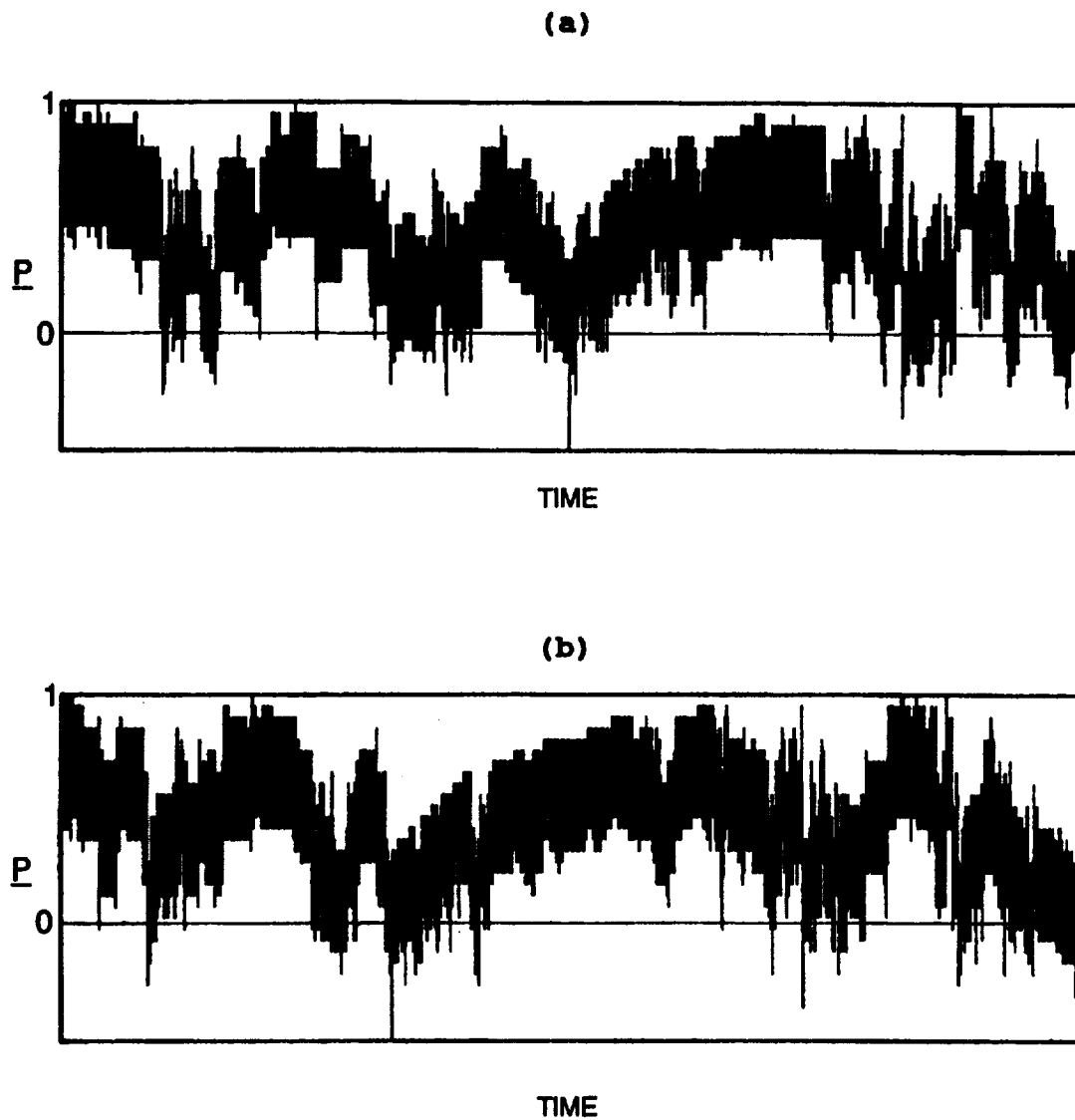


Figure 21 Reconstructions from Program JRECON. Reconstruction 3 (a) and Reconstruction 4 (b) contain more scrambling than reconstructions from RECON2.

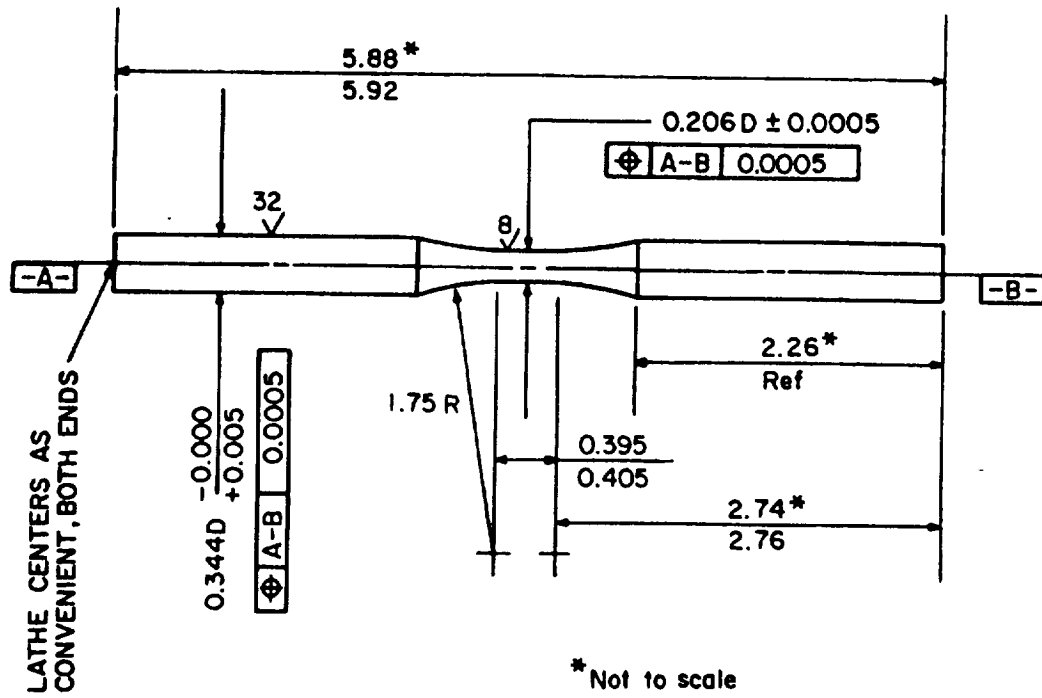


Figure 22 Smooth Specimen for Constant Amplitude Strain Tests.

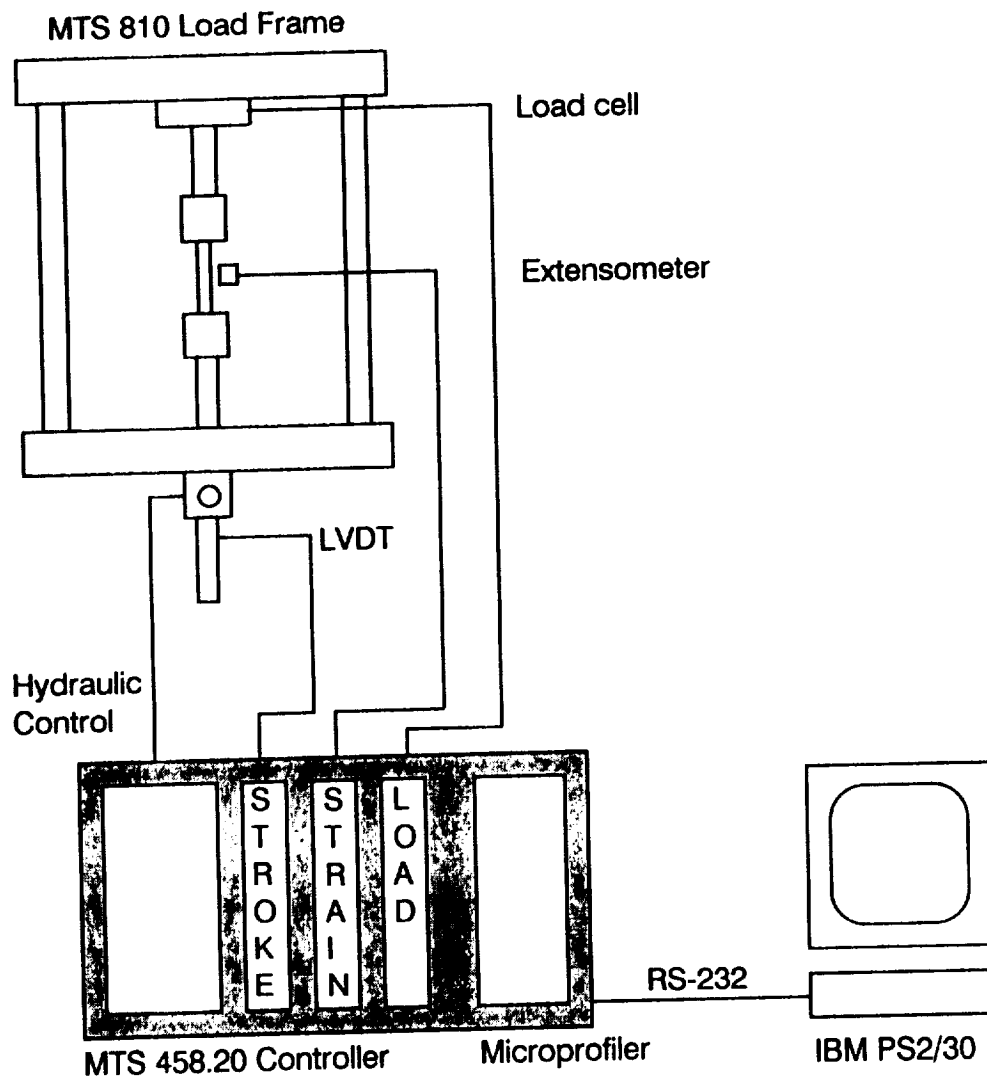


Figure 23 Hardware Configuration for Test Control.

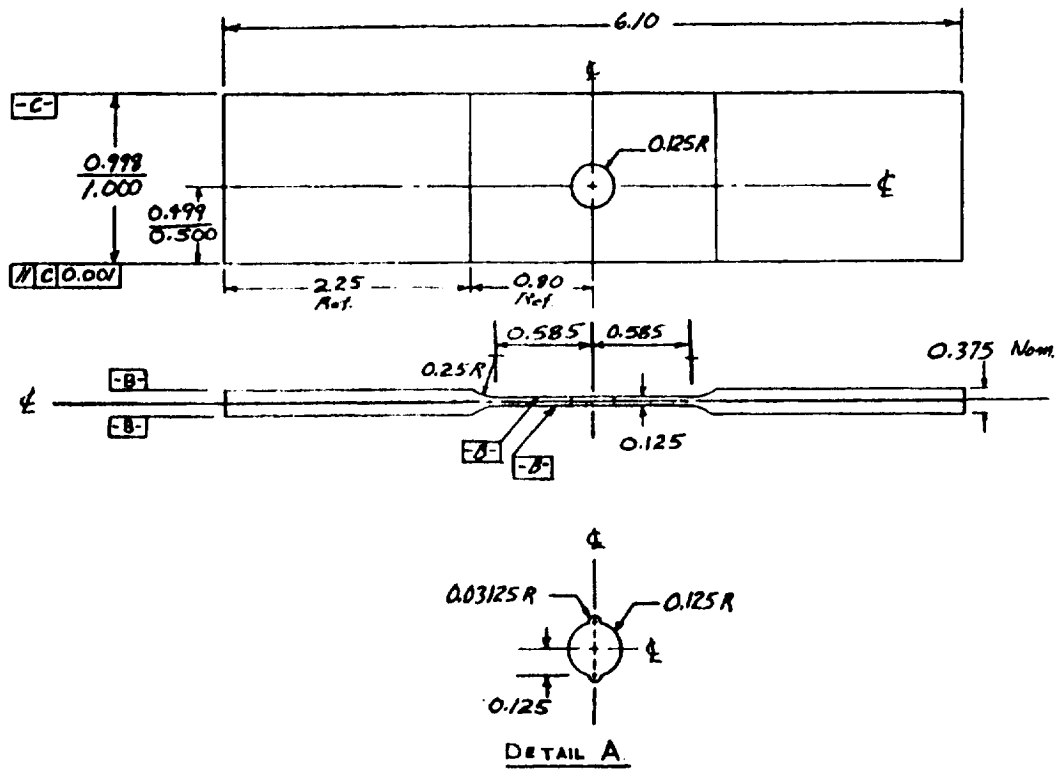


Figure 24 Notched Specimen for Variable Amplitude Load Testing.

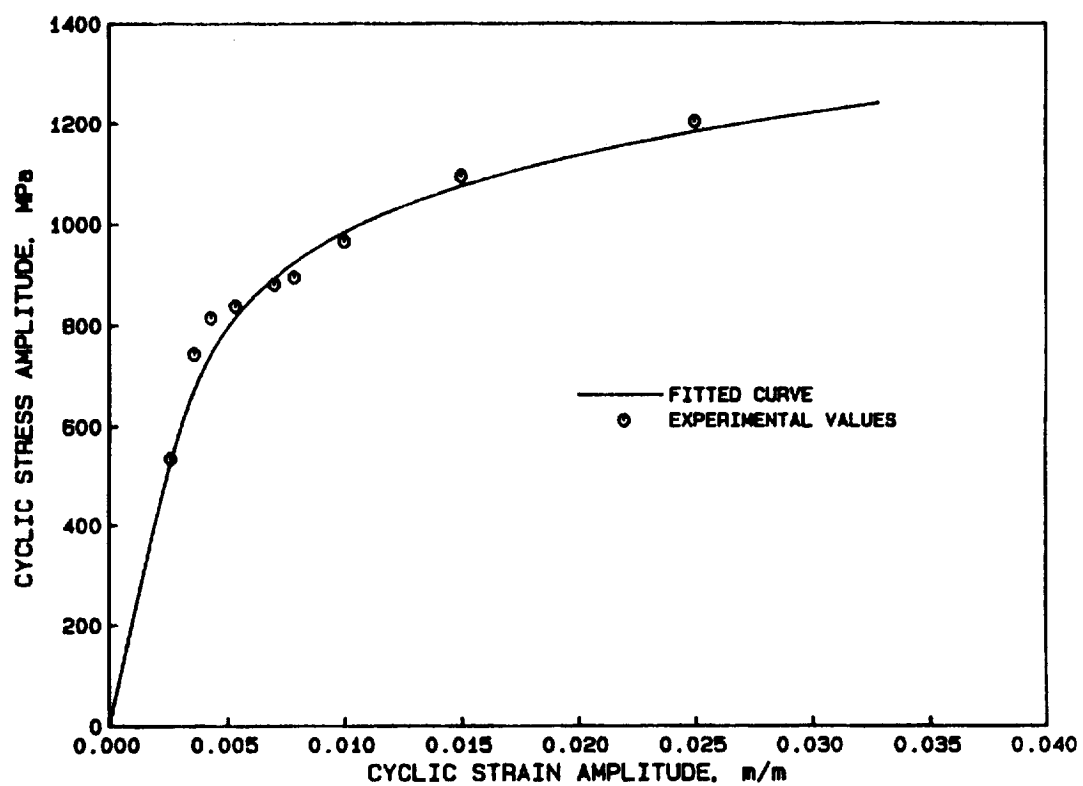


Figure 25 Cyclic Stress-Strain Curve for 4340 Steel with 1430 MPa Ultimate.

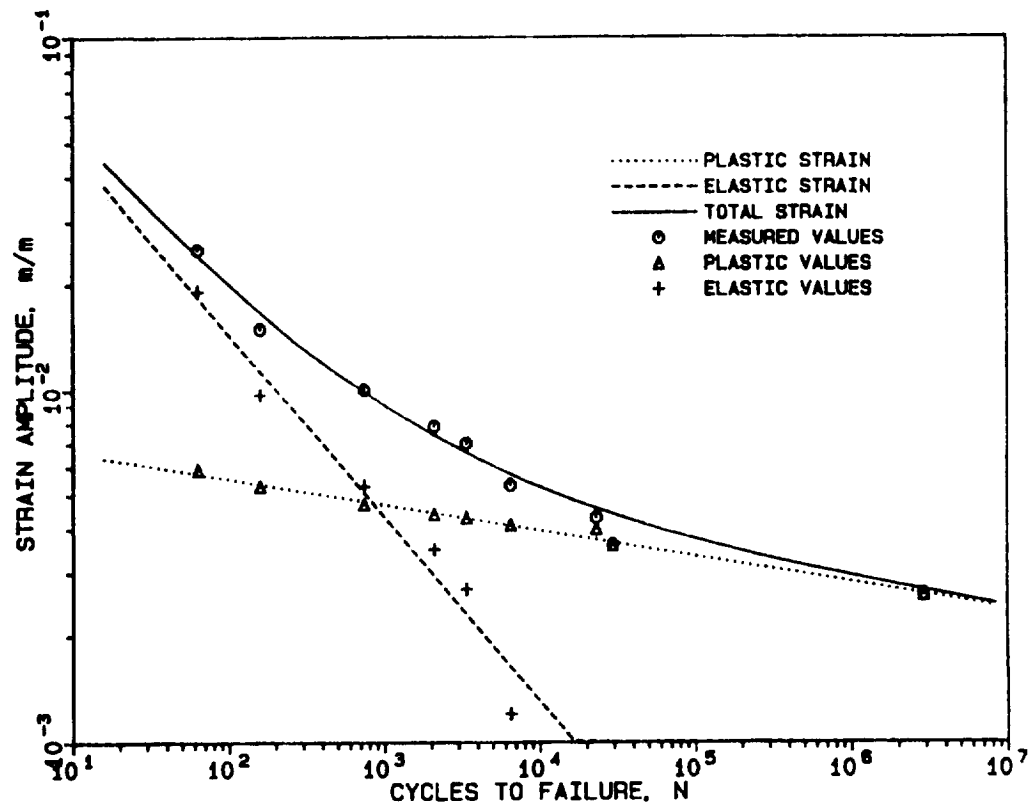


Figure 26 Strain-Life Curve for 4340 Steel with 1430 MPa Ultimate.

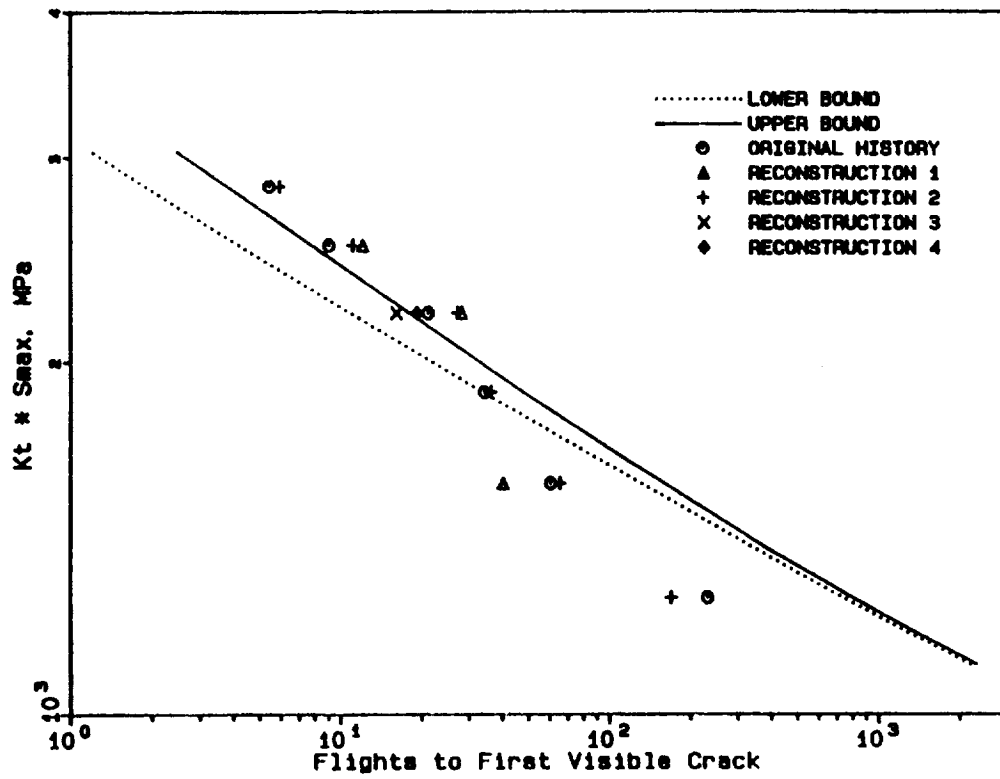


Figure 27 Flights to First Visible Crack for Notched Specimen.

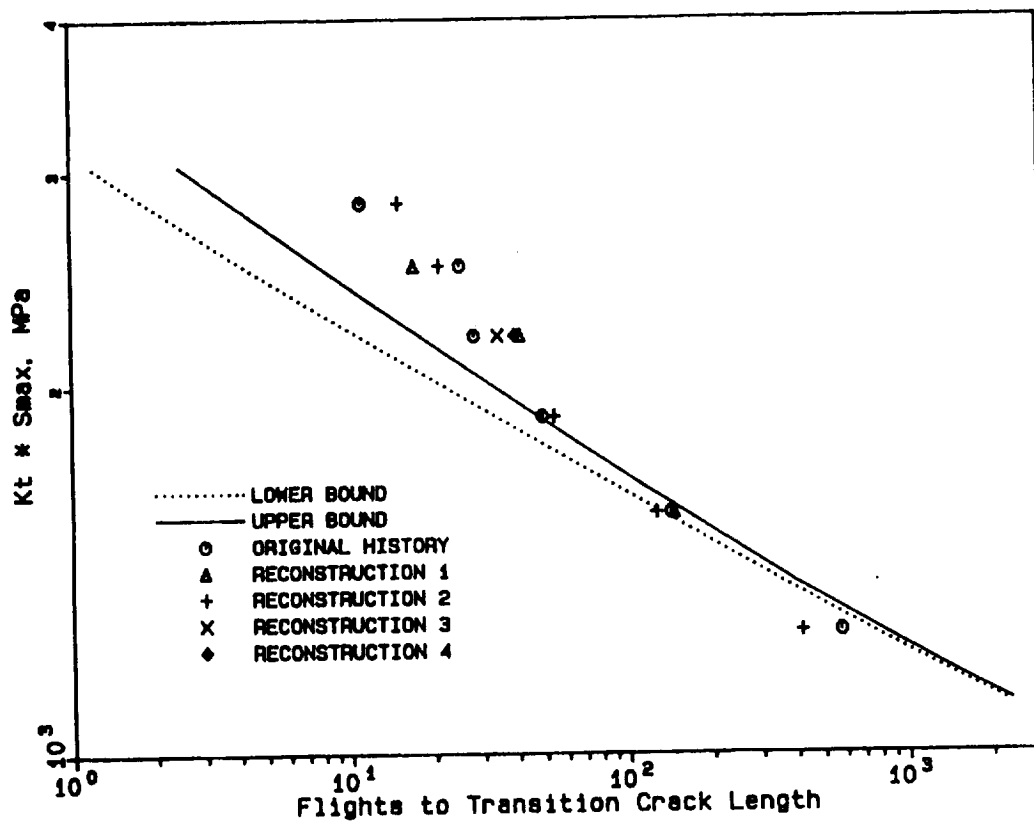


Figure 28 Flights to Transition Crack Length for Notched Specimen.

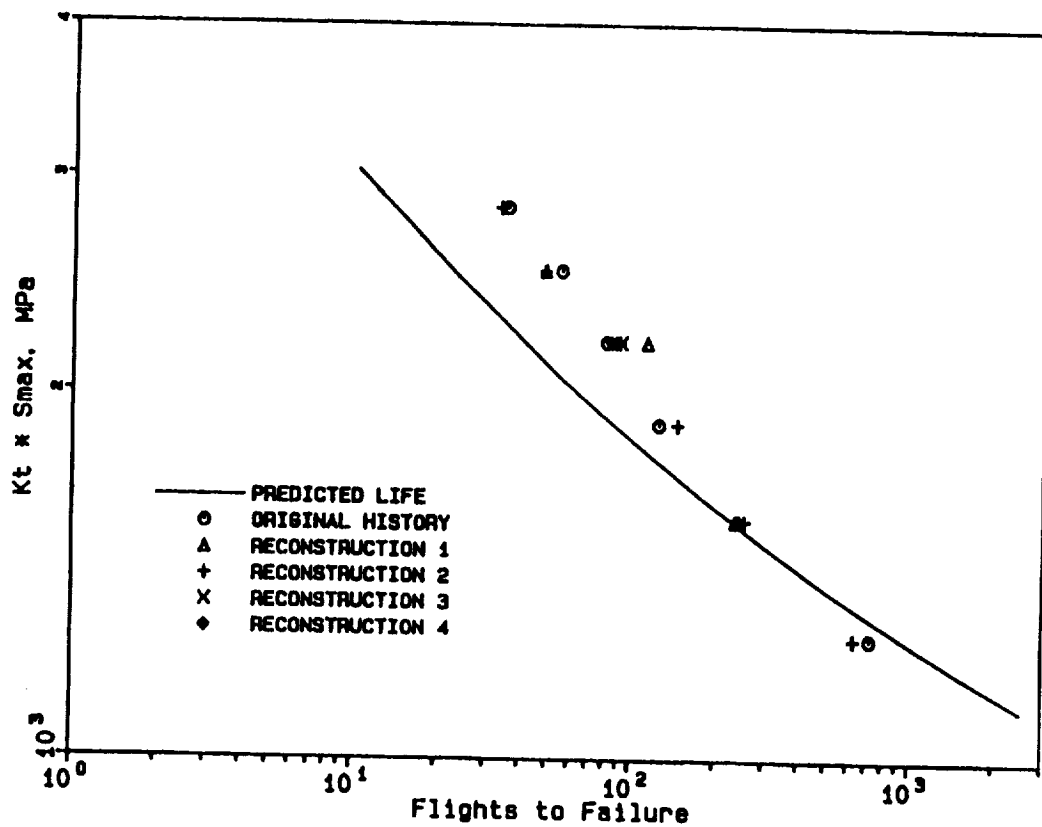


Figure 29 Flights to Failure for Notched Specimen.

Appendix A. Crack Growth Analysis

Computer Program

The FORTRAN program CRACK calculates the life of a cracked specimen subject to cycles dictated by a rain-flow matrix. The program goes through the rain-flow history and characterizes every load cycle and sorts the values. Then, the program performs numerical integration over the specified crack lengths. For each crack length along the integration, the sorted loads are searched until a value that produces a stress intensity, K , greater than ΔK_{th} is found. Only the load cycles with that value or greater are included in the crack growth computation for that particular crack length. The program then adds all the cycles using the inverse crack rate shown in equation 3.15. The required input is placed in two files. The first holds the geometry and material parameters shown in figure A.1a. The input values are as follows (according to variable type) in the same order as the input file:

Maximum Load, P	kips, real
Number of iterations	integer
Initial crack length, l_i	inches, real
Final crack length, l_f	inches, real
Walker coefficient, γ	real
Crack growth exponent, m	real
Crack growth coefficient, C	real

The second file contains the rain-flow matrix. The values are read across the rows and are shown in figure A.1b. Note that every value is read as a right justified two digit integer, so all the zeros are not necessary. If the matrix counts are high enough, the format must be changed to accommodate the larger size. The values for input may come from various sources.

The material properties used for input are shown in table 1. The load and number of iterations are chosen by the user to meet the specific application. Only the crack length values must be computed manually for the input. The initial crack length may either be zero or l_m , depending on the choice of the user. For the notched specimen used in testing, the difference in life is less than ten percent for the two choices. Figure A.1 shows l_i equal to zero for illustration. The final length depends on the mode of failure. If the specimen fails by brittle fracture due to K_{IC} being surpassed, then the crack length entered should be the corresponding crack length. If the specimen fails by gross section yielding, then the crack length that causes the net area to be sufficiently small for this condition is entered into the input. For the notched test specimen, the ductile failure crack length is used. This mode of failure is confirmed by the fracture surface of the specimen after failure being inclined 45 degrees to the loading, indicating a shear failure [25].

The program listing is as follows:

```

C234567 CRACK GROWTH PROGRAM
      DIMENSION IC(32,32),DSBAR(200)
      INTEGER PEAKS(200)
      REAL MAXLOAD,M,LIFE
      INTEGER COUNT,PKSTEMP

      PI=3.141593
      W=.5
      AGEOM=.15625
      DKTHRS=0.
      OPEN(UNIT=2,FILE='CRACK.DAT',TYPE='OLD')
      OPEN(UNIT=3,FILE='CRACK.PAR',TYPE='OLD')
      READ(3,*) MAXLOAD,NUMINCR,AI,AF,GAMMA,M,C
      AINCR=(AF-AI)/NUMINCR
      DO I=1,32
10      READ(2,10)(IC(I,J),J=1,32)
          FORMAT(32I2)
          DO K=1,32
              IF(IC(I,K).GT.0) THEN
                  IF(I.GT.K) THEN
                      MAX=I
                      MIN=K
                  ELSE
                      MAX=K
                      MIN=I
                  ENDIF
                  RMIN= 8.*MAXLOAD*(-.516+(MIN-1.)/31.*1.516)
                  RMAX= 8.*MAXLOAD*(-.516+(MAX-1.)/31.*1.516)
                  IF(RMAX.LE.0.) GOTO 30
                  COUNT=COUNT+1
                  IF(RMIN.GT.0.) THEN
                      R=RMIN/RMAX
                      DSBAR(COUNT)=RMAX*(1.-R)**GAMMA
                  ELSE
                      DSBAR(COUNT)=RMAX
                  ENDIF
                  PEAKS(COUNT)=IC(I,K)
                  NPEAKS=NPEAKS+IC(I,K)
30      ENDIF
          ENDDO
      ENDDO
C NOW SORT THE DSBAR MATRIX FROM LOW TO HIGH
      DO LAST=COUNT-1,1,-1
          DO I=1, LAST
              IF(DSBAR(I).GT.DSBAR(I+1)) THEN
                  DSTEMP=DSBAR(I)
                  PKSTEMP=PEAKS(I)
                  DSBAR(I)=DSBAR(I+1)
                  PEAKS(I)=PEAKS(I+1)
                  DSBAR(I+1)=DSTEMP
                  PEAKS(I+1)=PKSTEMP
              ENDIF
          ENDDO
      ENDDO
C MARCH THROUGH THE CRACK GROWTH. USE THE SORTED NORMALIZED LOAD
C MATRIX, DSBAR, IN CONJUNCTION WITH CRACK LENGTH AND REFERENCE LOAD
C TO DETERMINE WHERE K-TH IS SURPASSED. ONLY THE DELTA-K PASSED THAT
C POINT WILL BE USED IN THE COMPUTATION

```

```

      DO I=0,NUMINCR
        A=AINCR*I+AI+AGEOM
C      F=SQRT(1./COS((PI*A)/(2.*W)))      AGEOM IS INITIAL HOLE IN SPECIMEN
      FTOTAL=F*SQRT(PI*A)
      ISTART=1
C-----CHECK FOR LOWEST VALUE OF DELTA-K ABOVE THRESHOLD
      40  IF(ISTART.EQ.COUNT) THEN
          WRITE(6,50)
      50  FORMAT(' NO CRACK GROWTH...ALL CYCLES BELOW THRESHOLD')
          STOP
      ENDIF
      DKBAR=FTOTAL*DSBAR(ISTART)
      IF(DKBAR.LT.DKTHRS) THEN
          ISTART=ISTART+1
          GOTO 40
      ENDIF
C-----THE FIRST VALUE IS THE STARTING POINT FOR THIS ITERA-
C--TION. AS THE CRACK GROWS, MORE CYCLES WILL BE INCLUDED

      SUM=0.
      DO J=ISTART,COUNT
          SUM=SUM+PEAKS(J)*DSBAR(J)**M
      ENDDO
      DSEQV=(SUM/NPEAKS)**(1./M)
      DKEQV=FTOTAL*DSEQV
      DADN=C*DKEQV**M
      DNDA=1./DADN
      IF(1.GT.0) THEN
          AREA=AREA+(DNDAOLD-DNDA)/2.*AINCR
          AREA=AREA+DNDA*AINCR
      ENDIF
      DNDAOLD=DNDA
      ENDDO
      LIFE=AREA/NPEAKS
      WRITE(6,*) AREA,LIFE
      WRITE(6,*) NPEAKS
      END

```


Appendix B. Rain–Flow Reconstruction Program

The FORTRAN program JRECON is given in the following pages. The program is a modified version of the program RECON. An explanation of the input values is given in reference 45 and will not be repeated here. Most differences between JRECON and RECON2 do not change the program logic; rather, they are intended to clarify the program. The only statement that does change the logic is indicated with an arrow on page 90. In JRECON, the choice of NP=1 causes all cycles to be distributed individually, irrespective of the initial value of NOC. An example of a regenerated history is shown in figure A2 as a list of normalized load reversals.

```

C      THIS PROGRAM CONSTRUCTS A HISTORY USING THE R-F MATRIX
C      (CYCLE DIRECTIONS ARE CONSIDERED)
C
C      DATA LINE 1.  NP-A GIVEN ROW AND COLUMN HAS THIS VALUE OR
C                      LESS, THEN ALL PLACED IN ONE LOCATION.
C      DATA LINE 2.  NOC-THE NUMBER OF CYCLES FOR EACH ROW AND
C                      COLUMN, IF GREATER THAN NP, IS PLACED
C                      IN THIS MANY RANDOMLY CHOSEN LOCATIONS.
C      DATA LINE 3.  AB( , )-THE RAIN-FLOW MATRIX.
C                      (CYCLE DIRECTIONS CONSIDERED.)
C      NOTE: SIMPLEST FORM OF A RAIN-FLOW RECONSTRUCTED HISTORY
C            IS OBTAINED BY SETTING NP=100000
C
C      INTEGER A(32,32),KA(15024),P(15000),PF(15000),IL(15000)
4,    PE(15000),DUM,DUM1,DUM2,JL(15000),AB(32,32)
      DIMENSION R(20000)
      INTEGER STARTROW,STARTCOL,ROW,COL,POSITION
      DSEED=95173.D0
C
C      LEVEL IS THE NO. OF COLUMNS OR ROWS.
C
C      LEVEL=32
C
C      READ(5,*)NP
C      READ(5,*)NOC
C      DO 90 I=1,LEVEL
C      READ(5,*)(AB(I,J),J=1,LEVEL)
90    CONTINUE
C
C      RANDOM NUMBER GENERATION
C      NR=20000
C      CALL GGUBS(DSEED,NR,R)
C
C      REARRANGING THE R-F MATRIX
C
C      ISS=0
C      IB=1
C      ILA=LEVEL
C      P(1)=1
C      P(2)=LEVEL
C      PF(1)=1
C      PF(2)=LEVEL
C      AB(LEVEL,1)=0
C      AB(1,LEVEL)=1
C      MTOTAL=2
C
C      INCR=0
C      POSITION=0
C      DO 2 STARTROW=LEVEL,1,-1
C      COL=0
C      INCR=INCR+1
C      DO 1 ROW=STARTROW,LEVEL
C      COL=COL+1
C      POSITION=POSITION+1
C      IL(POSITION)=ROW
C      JL(POSITION)=COL
1    CONTINUE
      POSITION=POSITION+INCR

```

```

2      CONTINUE
C-----NOW DO UPPER HALF OF RAINFLOW MATRIX
      POSITION=1
      INCR=1
      DO 4 STARTCOL=LEVEL,2,-1
        ROW=1
        INCR=INCR+1
        DO 3 COL=STARTCOL,LEVEL
          POSITION=POSITION+1
          IL(POSITION)=ROW
          JL(POSITION)=COL
          ROW=ROW+1
3      CONTINUE
        POSITION=POSITION+INCR
4      CONTINUE

C      RECONSTRUCTION BEGINS
C
10     ITOTAL=LEVEL*(LEVEL-1)
      KCOUNT=1
      ISUM=2
9999   ISUM=ISUM+1
      KCOUNT=1
      IF(ISUM.GT.ITOTAL)GO TO 999
      L=0
      ILAST=IL(ISUM)
      J=JL(ISUM)
      IF(AB(ILAST,J).EQ.0)GO TO 9999
      IF(NP.EQ.1) NOC = AB(ILAST,J)
      NOC2=10000
      INUM=0
99     IF(INUM.EQ.NOC2)GO TO 9999
      IF(AB(ILAST,J).GT.NP) THEN
        INUM=INUM+1
        KNUM=AB(ILAST,J)/NOC
        A(ILAST,J)=KNUM
        NOC1=KNUM*NOC
        IF(NOC1.NE.AB(ILAST,J))THEN
          IF(INUM.EQ.NOC)A(ILAST,J)=KNUM+(AB(ILAST,J)-NOC1)
          NOC2=NOC
        ELSE
          NOC2=NOC
        END IF
      ELSE
        A(ILAST,J)=AB(ILAST,J)
        NOC2=INUM
      END IF
      IF(ILAST.LT.J)GO TO 300
      JM1=J-1
      ISU=ISUM-1
      DO 100 LI=1,ISU
        IPU=ILA-ILAST+1
        IDUM=IL(LI)
        JDUM=JL(LI)
        IF(A(IDUM,JDUM).EQ.0)GO TO 100
        IF(IL(LI).LT.JL(LI))THEN
          IF(IL(LI).LT.J.AND.JL(LI).GT.ILAST)GO TO 102
          GO TO 100
        END IF
        IF(IL(LI).GT.JL(LI))THEN
          IF(IL(LI).GT.ILAST.AND.JL(LI).LT.J)GO TO 102
          GO TO 100
        END IF

```

```

102      L=L+1
          KA(LI)=L
100      CONTINUE
710      IR=L*R(IB)
          KCOUNT=KCOUNT+1
          IB=IB+1
          NRAN=IR+1

C
C
C
          IF(KCOUNT.EQ.5)GO TO 711
776      IF(NRAN.EQ.1)THEN
C11      WRITE(6,*)NRAN,KCOUNT,ILAST,J,ISUM
711      KAN=0
98      DO 600 IA=1,MTOTAL
          IF(P(IA).EQ.1.AND.IA.EQ.1)GO TO 700
          IF(MTOTAL.LE.2)THEN
              IF(P(IA).EQ.1)GO TO 700
              END IF
              IF(P(IA).EQ.1.AND.P(IA+1).GT.LEVEL)THEN
                  IF(P(IA-1).GT.LEVEL)THEN
                      IF(P(IA-1).GT.116)GO TO 700
                      IAX=IA-1
                      IAMX=IA
7009      IAMX=IAMX-1
                      IF(IAMX.LT.1)GO TO 600
                      IF(P(IAMX).EQ.P(IA-1))GO TO 700
                      GO TO 7009
                      END IF
                      END IF
600      CONTINUE
700      IA3=IA+1
721      IF(P(IA3).GT.LEVEL)THEN
          IA3=IA3+1
          GO TO 721
          END IF
762      IF(P(IA3).LT.ILAST)THEN
          IA4=IA3-1
          IT=P(IA4)
          IA3=IA3+1
          DO 722 LE=IA3,MTOTAL
              IF(P(LE).EQ.IT)GO TO 723
              CONTINUE
722      IA1=LE
723      IA=LE
          LEE=LE+1
761      IF(LEE.GT.MTOTAL)GO TO 777
              IF(P(LEE).GT.LEVEL)THEN
                  LEE=LEE+1
                  GO TO 761
                  END IF
                  IA3=LEE
                  GO TO 762
                  END IF
777      IA1=IA
          ISS=ISS+A(ILAST,J)
724      KK=0
713      JI=IA
          DO 714 IM=1,JI
714      PP(IM)=P(IM)
          LLL=JI+1
          PP(LLL)=(ILAST*100)+J
          JK=JI+(A(ILAST,J)*2)+1

```

```

JE=JI+2
DO 701 IMM=JE,JK,2
PP(IMM)=ILAST
IM1=IMM+1
PP(IM1)=J
701 CONTINUE
MTOTAL=MTOTAL+(A(ILAST,J)*2)+2
IM1=IM1+1
PP(IM1)=(ILAST*100)+J
IM1=IM1+1
J11=JI
DO 702 IB=IM1,MTOTAL
J11=J11+1
PP(IB)=P(J11)
702 CONTINUE
DO 703 IN=1,MTOTAL
703 P(IN)=PP(IN)
GO TO 99
END IF
IF(KCOUNT.EQ.5)GO TO 99

C
C
C

K=1
DO 200 LI=1,ISU
IPU=ILA-ILAST+1
IDUM=IL(LI)
JDUM=JL(LI)
IF(A(IDUM,JDUM).EQ.0)GO TO 200
IF(IL(LI).LT.JL(LI))THEN
IF(IL(LI).LT.J.AND.JL(LI).GT.ILAST)GO TO 201
GO TO 200
END IF
IF(IL(LI).GT.JL(LI))THEN
IF(IL(LI).GT.ILAST.AND.JL(LI).LT.J)GO TO 201
GO TO 200
END IF
201 K=K+1
KA(I)=K
IF(NRAN.EQ.K)GO TO 500
200 CONTINUE
500 I=IL(LI)
II=JL(LI)
ISS=A(ILAST,J)+ISS
IF(I.EQ.1.AND.II.EQ.LEVEL)THEN
NRAN=1
GO TO 776
END IF
IF(I.LT.II)THEN
IF(I.GE.J)GO TO 710
END IF
CALL SORT (I,II,ILAST,J,LK,KC,P,PP,MTOTAL,LEVEL,A)
IF(I.EQ.1.AND.II.EQ.LEVEL)GO TO 98
GO TO 99

C
C
C
300 ISU=ISUM-1
DO 800 LB=1,ISU
IPU=ILA-ILAST+1
IDUM=IL(LB)
JDUM=JL(LB)
IF(A(IDUM,JDUM).EQ.0)GO TO 800

```

```

      IF(IL(LB).GT.JL(LB))THEN
      IF(IL(LB).GT.J.AND.JL(LB).LT.ILAST)GO TO 801
      GO TO 800
      ELSE
      IF(IL(LB).LT.ILAST.AND.JL(LB).GT.J)GO TO 801
      GO TO 800
      END IF
801      L=L+1
      KA(I)=L
800      CONTINUE
      KCOUNT=1
1711      IR=L+R(IB)
      KCOUNT=KCOUNT+1
      IB=IB+1
      NRAN=IR+1
      IF(KCOUNT.EQ.5)GO TO 1712

C
C
C
1712      IF(NRAN.EQ.1)THEN
101      NAM=0
      IC=A(ILAST,J)
      DO 1901 IA=1,MTOTAL
      IF(P(IA).EQ.1.AND.IA.EQ.1)GO TO 1916
      IF(MTOTAL.LE.2)THEN
      IF(P(IA).EQ.1)GO TO 1916
      END IF
      IF(P(IA).EQ.1.AND.P(IA+1).GT.LEVEL)THEN
      IF(P(IA-1).GT.LEVEL)THEN
      IF(P(IA-1).GT.116)GO TO 1916
      IAMX=IA-1
7010      IAMX=IAMX-1
      IF(IAMX.LT.1)GO TO 1901
      IF(P(IAMX).EQ.P(IA-1))GO TO 1916
      GO TO 7010
      END IF
      END IF
1901      CONTINUE
1916      IF(IA.EQ.1)GO TO 2103
      IAM1=IA-1
      PIAM1=P(IAM1)
      IAM2=IA-1
2101      IAM2=IAM2-1
      IF(P(IAM2).NE.PIAM1)GO TO 2101
2100      IAM1=IAM2+1
2108      IF(P(IAM1).LT.ILAST)THEN
      IAI=IAM2
      IA=IA1
      IAM3=IAM2-1
2105      IF(IAM3.LE.0)GO TO 2103
      IF(P(IAM3).GT.LEVEL)GO TO 2104
      IAM3=IAM3-1
      GO TO 2105
      IAM4=IAM3-1
2104      IF(IAM4.LE.0)GO TO 2103
2107      IF(P(IAM4).EQ.P(IAM3))GO TO 2106
      IAM4=IAM4-1
      GO TO 2107
2106      IAM1=IAM4+1
      IAM2=IAM4
      GO TO 2108
      END IF
2103      IAI=IA

```

```

2102      JI=IA
          JIM1=JI-1
          IF(JI.EQ.1)GO TO 1909
          DO 1910 I=1,JIM1
1910      PP(I)=P(I)
1909      JK=JI+(IC*2)-1
          PP(JI)=(ILAST*100)+J
          JE=JI+1
          DO 1911 I=JE,JK,2
          PP(I)=ILAST
          IP=I+1
          PP(IP)=J
1911      CONTINUE
          MTOTAL=MTOTAL+(IC*2)+2
          IP=IP+1
          PP(IP)=(ILAST*100)+J
          IP=IP+1
          J11=JI-1
          DO 1912 IB=IP,MTOTAL
          J11=J11+1
          PP(IB)=P(J11)
1912      CONTINUE
          DO 1914 KB=1,MTOTAL
1914      P(KB)=PP(KB)
          GO TO 99
          END IF
          IF(KCOUNT.EQ.5)GO TO 99

C
C
C

          K=0
          DO 900 LB=1,ISU
          IPU=ILA-ILAST+1
          IDUM=IL(LB)
          JDUM=JL(LB)
          IF(A(IDUM,JDUM).EQ.0)GO TO 900
          IF(IL(LB).GT.JL(LB))THEN
          IF(IL(LB).GT.J.AND.JL(LB).LT.ILAST)GO TO 901
          GO TO 900
          ELSE
          IF(IL(LB).LT.ILAST.AND.JL(LB).GT.J)GO TO 901
          GO TO 900
          END IF
          K=K+1
901      KA(I)=K
          IF(NRAN.EQ.K)GO TO 1200
          CONTINUE
900      I=IL(LB)
1200      II=JL(LB)
          ISS=ISS+A(ILAST,J)
          CALL SORT (I,II,ILAST,J,LK,KC,P,PP,MTOTAL,LEVEL,A)
          IF(I.EQ.1.AND.II.EQ.LEVEL)GO TO 101
          GO TO 99
999      MT=0
          WRITE(6,*)MTOTAL
          DO 1001 I=1,MTOTAL
          IF(PP(I).GT.LEVEL)GO TO 1001
          MT=MT+1
          PE(MT)=PP(I)
1001      CONTINUE
          WRITE(6,*)MT
          WRITE(6,1300)(PE(I),I=1,MT)
1300      FORMAT(1X,11(15,1X))

```

```

      STOP
      END
      SUBROUTINE SORT(I,II,ILAST,J,LK,KC,P,PP,MTOTAL,LEVEL,A)
      INTEGER PP(15000),A(32,32),P(15000)
      IF(I.EQ.1.AND.II.EQ.LEVEL)GO TO 200
      LEVEL=32
      IP=(I*100)+II
C
C
C      IF(I.GT.II)THEN
C
C      IF(ILAST.GT.J)THEN
      DO 1 L=1,MTOTAL
      LP1=L+1
1      IF(P(L).EQ.II.AND.P(LP1).EQ.IP)GO TO 2
2      LPL=LP1
500    LPL=LPL+1
      IF(P(LPL).GT.LEVEL)GO TO 500
      LPL1=LPL
501    LPL1=LPL1+1
      IF(P(LPL1).GT.LEVEL)GO TO 501
      IF(P(LPL).LT.ILAST.AND.P(LPL1).LT.J)THEN
      I=1
      II=LEVEL
      GO TO 200
      END IF
      DO 3 LK=LP1,MTOTAL
3      IF(P(LK).LE.LEVEL)GO TO 4
4      IF(P(LK).GT.ILAST)GO TO 45
      LK1=LK-1
48     DO 410 LK2=LK,MTOTAL
410    IF(P(LK2).EQ.P(LK1))GO TO 42
42     LK3=LK2
      LK3=LK2+1
      IF(P(LK3).LE.LEVEL)GO TO 43
      LK4=LK3
      LK4=LK4+1
      IF(P(LK4).GT.ILAST)GO TO 44
      LK1=LK3
      LK=LK4
      GO TO 48
43     IF(P(LK3).EQ.LEVEL)THEN
      LP1=LK3-1
      END IF
      GO TO 45
44     LP1=LK3-1
45     DO 5 LI=1,LP1
5      PP(LI)=P(LI)
      IPM1=LP1+1
      PP(IPM1)=(ILAST*100)+J
      IPM2=IPM1+1
      K=A(ILAST,J)
      KK=1
      L=IPM2
6      PP(L)=ILAST
      L=L+1
      PP(L)=J
      IF(K.EQ.KK)GO TO 8
      KK=KK+1
      L=L+1
      GO TO 6
8      L=L+1
      PP(L)=(ILAST*100)+J

```



```

L=L+1
L2=IPM1
DO 9 IK=L2,MTOTAL
PP(L)=P(IK)
L=L+1
9 CONTINUE
MTOTAL=L-1
ELSE
C
C
DO 10 JI=2,MTOTAL
JI1=JI-1
10 IF(P(JI).EQ.I.AND.P(JI1).EQ.IP)GO TO 11
11 JIPI=JI+1
DO 12 LE=JIP1,MTOTAL
12 IF(P(LE).EQ.P(JI1))GO TO 13
13 LE=LE-2
IF(P(LE).GT.LEVEL)GO TO 14
JI=LE
GO TO 16
LEE=LE
14 LEE=LEE-1
140 IF(P(LEE).NE.P(LE))GO TO 140
LE1=LEE+1
150 IF(P(LE1).GT.ILAST)THEN
DO 17 LC=LE1,MTOTAL
LC1=LC+1
17 IF(P(LC).EQ.P(LE))GO TO 18
18 JI=LC
ELSE
LE2=LEE-1
220 IF(P(LE2).LE.LEVEL)GO TO 190
LE3=LE2
210 LE3=LE3-1
IF(P(LE3).EQ.P(LE2))GO TO 180
GO TO 210
LE4=LE3+1
180 IF(P(LE4).LT.ILAST)THEN
LEE=LE3
GO TO 220
ELSE
JI=LE2
END IF
GO TO 16
190 IF(P(LE2).EQ.I)THEN
JI=LE2
END IF
END IF
16 DO 19 IK=1,JI
19 PP(IK)=P(IK)
ICC=A(ILAST,J)*2
IK=0
IC=JI+1
PP(IC)=(ILAST*100)+J
20 IC=IC+1
IK=IK+2
PP(IC)=ILAST
IC=IC+1
PP(IC)=J
IF(IK.EQ.ICC)GO TO 21
GO TO 20
21 MTOTAL=MTOTAL+ICC+2
IC=IC+1

```

```

PP(IC)=(ILAST*100)+J
IC=IC+1
IA=JI
DO 22 ID=IC,MTOTAL
IA=IA+1
22 PP(ID)=P(IA)
END IF
END IF
IF(I.GT.II)GO TO 2000

C
C
C
IF(I.LT.II)THEN
IF(ILAST.LT.J)THEN
DO 23 KI=1,MTOTAL
KI=KI+1
23 IF(P(KI).EQ.II.AND.P(KI1).EQ.IP)GO TO 24
24 DO 25 KC=KI1,MTOTAL
25 IF(P(KC).LE.LEVEL)GO TO 250
250 IF(P(KC).LT.ILAST)GO TO 231
KCI=KC-1
DO 233 KB=KC,MTOTAL
233 IF(P(KB).EQ.P(KCI))GO TO 234
234 KI=KB
GO TO 24
231 KTRY=KI+1
IF(KC.EQ.KTRY)THEN
LIP=KC-1
ELSE
LIP=KC-2
END IF
IP1=LIP+1
DO 26 KB=1,LIP
26 PP(KB)=P(KB)
PP(IP1)=(ILAST*100)+J
K=A(ILAST,J)
L=IP1+1
DO 27 KK=1,K
PP(L)=ILAST
L=L+1
PP(L)=J
L=L+1
27 CONTINUE
PP(L)=(ILAST*100)+J
L=L+1
L2=IP1
DO 29 IK=L2,MTOTAL
PP(L)=P(IK)
L=L+1
29 CONTINUE
MTOTAL=L-1
ELSE

C
C
DO 30 NI=2,MTOTAL
NI1=NI-1
30 IF(P(NI1).EQ.IP.AND.P(NI).EQ.I)GO TO 31
31 DO 32 NC=NI,MTOTAL
NC1=NC+1
32 IF(P(NC).EQ.II.AND.P(NC1).EQ.IP)GO TO 333
333 NC1=NC1-2
GO TO 41
33 NC1=NC-2
41 IF(P(NC1).LE.LEVEL)GO TO 300

```

```

320      NC2=NC1
        NC2=NC2-1
        IF(P(NC2).EQ.P(NC1))GO TO 310
        GO TO 320
310      NC3=NC2+1
        IF(P(NC3).LT.ILAST)GO TO 330
        NC=NC3
        GO TO 33
300      IF(P(NC1).EQ.I)THEN
        JI=NC1
        END IF
        GO TO 380
330      JI=NC1
380      DO 37 IE=1,JI
37      PP(IE)=P(IE)
        PP(IE)=(ILAST*100)+J
        K=A(ILAST,J)
        KK=1
        L=IE+1
39      PP(L)=ILAST
        L=L+1
        PP(L)=J
        IF(K.EQ.KK)GO TO 38
        KK=KK+1
        L=L+1
        GO TO 39
38      L=L+1
        PP(L)=(ILAST*100)+J
        L=L+1
        L2=JI+1
        DO 40 IS=L2,MTOTAL
        PP(L)=P(IS)
        L=L+1
40      CONTINUE
        MTOTAL=L-1
        END IF
2000     DO 1000 K=1,MTOTAL
1000     P(K)=PP(K)
200     RETURN
        END

```

Appendix C. Constant Amplitude Strain Control Program

The constant amplitude strain tests are controlled by the BASIC program CONAMP. The parameters to control frequency and strain range are input interactively. The program will cycle the specimen at the desired frequency until a specified count is reached. The test then slows to two hertz so that an X-Y recorder may be used to capture the material stress-strain response. The cycles at which the program slows may be adjusted to the length of the test. This provides for a reasonable sampling rate of the data. The data may then be extracted from the X-Y recordings to fit to the desired equation. This program listing is as follows:

```

10  CLS
20  I% = 0 : ' I IS A COUNTING VARIABLE
30  NUM.LOOPS = 10 : 'INITIAL NUMBER OF LOW FREQUENCY LOOPS
40  NL = 0
50  DIM CYCLES(5)
60  INPUT "          FULL SCALE STRAIN (IE +/- X)";FULL.SCALE.STRAIN
70  INPUT "          DESIRED STRAIN AMPLITUDE FOR TEST";STRAIN.AMPLITUDE
100  SA = STRAIN.AMPLITUDE / FULL.SCALE.STRAIN*100
110  IF SA <= 100 THEN 140
120  PRINT "INVALID STRAIN AMPLITUDE, PLEASE TRY AGAIN"
130  GOTO 70
140  '
150  INPUT "STEADY STATE TEST FREQUENCY(.001<FREQ<80)";FREQ
160  INPUT "          ANTICIPATED LIFE";PRED.LIFE
170  PL = INT(PRED.LIFE/10)
180  IF PL > 999999! THEN PL=999999!
190  CYCLES(1) = 50 : CYCLES(2) = 50
200  CYCLES(3) = 90 : CYCLES(4) = 90
210  CYCLES(5) = PL
220  OPEN "COM1:9600,E,7,,CS,DS,PE" AS #1
230  PRINT #1,"100R"
240  INPUT #1,A
250  PRINT #1,"100S"
260  INPUT #1,A
270  PRINT #1,"1T"
280  INPUT #1,A
290  PRINT #1,"1B"
300  INPUT #1,A
310  PRINT #1,"I"
320  INPUT #1,A
330  LOCATE 5,27
340  PRINT "BYTES AVAILABLE: ";A
350  ' DO FIRST NUM.LOOPS LOOPS AT LOW FREQUENCY TO PLOT ON X-Y RECORDER
360  PRINT #1,".1K"
370  INPUT #1,A
380  PRINT #1,SA,"H"
390  INPUT #1,A
400  TOT.COUNT = TOT.COUNT + NUM.LOOPS + 1
410  PRINT #1,".1K"
420  INPUT #1,A
430  PRINT #1,-SA,"H"
440  INPUT #1,A
450  PRINT #1,".1K"
460  INPUT #1,A
470  PRINT #1,"P"
480  INPUT #1,A
490  PRINT #1,NUM.LOOPS,"F"
500  INPUT #1,A
520  PRINT #1,SA,"H"
530  INPUT #1,A
540  IF I% = 5 THEN 640
550  '
560
570
580
600  NUM.LOOPS = 10
610  I% = I% + 1
620  NC = CYCLES(I%)-10
640  PRINT #1,FREQ,"K"
650  INPUT #1,A

```

```

*** THIS BLOCK STARTS THE TEST ***
IF I%>0 THEN 610
PRINT #1,"J"
INPUT #1,A

```

```

660 PRINT #1,"P"
670 INPUT #1,A
680 PRINT #1,SA,"H"
690 INPUT #1,A
700 PRINT #1,FREQ,"K"
710 INPUT #1,A
713 PRINT #1,"P"
716 INPUT #1,A
720 PRINT #1,NC,"F"
730 INPUT #1,A
740 PRINT #1,-SA,"H"
750 INPUT #1,A
760 'AT THIS POINT, HIGH FREQUENCY BLOCK IS LOADED,
770 'BUT MONITER COUNT TO ALLOW TESTING MACHINE TO COMPLETE LOW
780 'FREQUENCY CYCLES. THE PROGRAM AND TESTING STAY CLOSE THIS WAY.
790 PRINT #1,"L"
800 INPUT #1,SEGMENT.COUNT
802 CURRENT.COUNT = INT((SEGMENT.COUNT - 2)/2)
803 LOCATE 10,20
806 PRINT CURRENT.COUNT;" CYCLES COMPLETED"
810 IF CURRENT.COUNT < TOT.COUNT-1 THEN 790
820 LOCATE 13,20
830 PRINT"TEST BACK AT HIGH FREQUENCY"
840 ' MONITER COUNT TO FOREWARN OF SLOWDOWN
845 TOT.COUNT = TOT.COUNT + NC : '<--- UPDATE COUNT
850 PRINT #1,"L"
860 INPUT #1,SEGMENT.COUNT
865 CURRENT.COUNT = INT((SEGMENT.COUNT - 2)/2)
870 LOCATE 10,20
880 PRINT CURRENT.COUNT;" CYCLES COMPLETED"
890 IF CURRENT.COUNT < TOT.COUNT - 30 THEN 850
910 BEEP
920 LOCATE 13,20
930 PRINT "GET READY....TEST SLOW-DOWN FOR X-Y RECORDER"
940 GOTO 400
950 END

```

Appendix D. Variable Amplitude Load Control Program

The variable amplitude load tests are controlled by the BASIC program SPEC11, given in the following two pages. The normalized load history is read from a data file in the form of figure A.2. Note, from lines 260- 290 of the program, that the data is read in rows of ten values. These lines must be changed if some other number is used. Also, if the number of reversals in the history is not 1020, lines in the program code, in addition to the input data file, must be changed. Since the first value in the history is the same as the last value, the number of reversals plus one is the actual number used in the data file. Line numbers 210, 250, 300, 393, 394, and 590 require changing to accomodate a different sized history. These constants may be replaced with one variable to make the program more flexible. Finally, in lines 670 and 690, the range of i multiplied by the range of j must be equal to the number of reversals in the history. This condition must be met to ensure that every reversal is executed.

The control parameters: load cartridge range, scaling load for the history, and the input filename, are input interactively during runtime. If the program is run as shown, simply entering the three parameters during runtime will be sufficient, assuming the data file is the appropriate size. Otherwise, the aforementioned changes must be made to the program and data file.

```

5      CLS
10     LOCATE 15,15 : PRINT"RESET COUNTER ON MICROCONSOLE"
20     LOCATE 16,15 : PRINT"PRESS <F5> KEY WHEN READY"
30     LOCATE 24,1 : STOP
40     CLS
50     PRINT"INPUT DATA:"
55     PRINT
60     INPUT"LOAD CARTRIDGE RANGE (IE +/- X KIPS; ENTER X)";LOAD.RANGE
65     INPUT"          MAXIMUM NOMINAL LOAD(IN KIPS)";MAX.LOAD
70     INPUT"          DATA FILE NAME";FILENAME$
80     FULL.SWING.FREQ = .53
120    IF MAX.LOAD < LOAD.RANGE THEN 150
125    LOCATE 5,16
130    PRINT"MAXIMUM NOMINAL STRESS IS TOO HIGH, PLEASE TRY LOWER VALUE"
140    GOTO 90
150    TOTAL.TIME = 0 : MAX.FREQ = 20 'INITIALIZE VARIABLES
155    OPEN "COM1:9600,E,7,,CS,DS,PE" AS #1
160    IF ZAP=1 THEN 430
180    OPEN FILENAME$ FOR INPUT AS #2 'SETS THE INPUT FILE
190    'READ NORMALIZED PEAK DATA
200    INPUT #2,NUMBER.POINTS
210    IF NUMBER.POINTS <> 1021 THEN GOSUB 1020
220    DIM A(NUMBER.POINTS+24) 'A IS ACTUAL PEAKS MATRIX
230    DIM P(NUMBER.POINTS) 'P IS NORMALIZED PEAKS MATRIX
240    DIM F(NUMBER.POINTS+24) 'F IS FREQUENCY MATRIX
250    NUMBER.ROWS = 102
255    LOCATE 7,16 : PRINT"LAST ROW READ: "
260    FOR I=0 TO NUMBER.ROWS-1
265    LOCATE 7,30 : PRINT I+1
270    J=I*10 'FOR 10 POINTS PER ROW
280    INPUT #2,P(J+1),P(J+2),P(J+3),P(J+4),P(J+5),
        P(J+6),P(J+7),P(J+8),P(J+9),P(J+10)
290    NEXT I
295    LOCATE 7,16 : PRINT USING"#### PEAKS READ FROM FILE";NUMBER.POINTS
300    P(1021) = P(1)
310    TIME.CONSTANT = 2*FULL.SWING.FREQ 'THIS VARIABLE IS USED FOR SCALING
320    'FREQUENCIES TO MATCH AMPLITUDES
325    LOCATE 8,16 : PRINT"PEAK BEING PROCESSED: "
330    FOR I=1 TO NUMBER.POINTS-1
332    LOCATE 8,39 : PRINT USING"####";I
340    LOAD.AMPLITUDE = ABS(P(I+1)-P(I)) 'NOTE: STILL NORMALIZED
350    F(I) = TIME.CONSTANT/LOAD.AMPLITUDE
360    IF F(I) > MAX.FREQ THEN F(I)=MAX.FREQ
370    TOTAL.TIME = TOTAL.TIME + 1/F(I)
380    A(I) = P(I+1)*MAX.LOAD
390    NEXT I
392    FOR M=1 TO 25
393    A(1020+M) = A(M)
394    F(1020+M) = F(M)
395    NEXT M
398    LOCATE 8,16 : PRINT"ALL POINTS PROCESSED FOR MICROPROFILER INPUT"
400    TT = INT(TOTAL.TIME/2)
410    LOCATE 9,15 : PRINT TT;" SECONDS TO EXECUTE ONE FLIGHT"
420    'LOAD & FREQUENCY MATRICES ARE NOW SET, PROCEED TO MICROPROFILER COMMANDS
430    PRINT #1,"100R" 'SET RETURN RATE
440    INPUT #1,Z
450    PRINT #1,LOAD.RANGE,"S" 'SET SCALE FACTOR TO LOAD RANGE
460    INPUT #1,A
470    PRINT #1,"1T" 'SET TIME BASE TO 1 SECOND
480    INPUT #1,A
490    PRINT #1,"1B" 'LOW RESOLUTION HAVERSINE (126 BYTES)
500    INPUT #1,A
510    PRINT #1,"I" 'RESET THE BUFFER
520    INPUT #1,NUM.BYTES
530    IF ZAP=1 THEN 620
540    MAX.SEGMENTS = INT(NUM.BYTES/126)
545    LOCATE 10,15

```



```

550 PRINT MAX.SEGMENTS;" SEGMENTS ALLOWED; USING 255 EACH SEND"
560 'TAKE CARE OF INITIAL RAMP & SHIFT OF PEAKS
570 PRINT #1, ".2K"
580 INPUT #1,A
590 PRINT #1,A(1020),"H"
600 INPUT #1,A
602 FOR N=1 TO 25
603 PRINT #1,F(N),"K"
604 INPUT #1,A
605 PRINT #1,"P"
606 INPUT #1,A
607 PRINT #1,A(N),"H"
608 INPUT #1,A
609 NEXT N
610 TOTAL.SEGMENTS = 26
612 LOCATE 12,16 : PRINT"LAST LOADED PEAK: "
614 LOCATE 13,16 : PRINT"LAST EXECUTED PEAK: SEE MICROCONSOLE COUNTER"
617 GOTO 650
620 PRINT #1,"5N"
630 INPUT #1,A
640 TOTAL.SEGMENTS = 0
650 PRINT #1,"J"
660 INPUT #1,A
670 FOR I=0 TO 3
680 TOTAL.SEGMENTS = TOTAL.SEGMENTS+255
690 FOR J=1 TO 255
700 K=I*255+J+25
710 PRINT #1,F(K),"K"
720 INPUT #1,A
730 PRINT #1,"P"
740 INPUT #1,A
750 PRINT #1,A(K),"H"
760 INPUT #1,A
770 LOCATE 12,36 : PRINT K+NUMBER.FLIGHTS*1020
780 NEXT J
790 'TRAP THE MICROPROFILER COUNT SO AS NOT TO OVERFLOW ITS MEMORY
800 PRINT #1,"L"
810 INPUT #1,CURRENT.COUNT
820 IF CURRENT.COUNT<TOTAL.SEGMENTS-25 THEN 800
830 NEXT I
840 NUMBER.FLIGHTS = NUMBER.FLIGHTS+1
850 LOCATE 15,15
860 PRINT NUMBER.FLIGHTS;" FLIGHTS COMPLETED"
870 PRINT #1,"L"
880 INPUT #1,COUNT
890 IF COUNT < TOTAL.SEGMENTS THEN 870
910 CLOSE #1 : ZAP=1
920 BEEP
930 LOCATE 20,10 : PRINT"COUNTDOWN TO NEXT CYCLE"
940 FOR Q=1 TO 1000
950 LOCATE 20,34 : PRINT USING"###";1000-Q
960 NEXT Q
970 BEEP
980 LOCATE 20,10 : PRINT"
990 GOTO 155
1000 '
1010 '
1020 PRINT"THIS PROGRAM IS DESIGNED FOR AN INPUT FILE CONSISTING OF A"
1030 PRINT"VALUE INDICATING NUMBER OF PEAKS IN HISTORY, FOLLOWED BY THE"
1040 PRINT"ACTUAL PEAK VALUES IN ROWS OF 10. IF YOUR INPUT FILE IS DIFFERENT,"
1050 PRINT"ADJUSTMENTS MUST BE MADE TO INPUT LINES IN PROGRAM."
1060 END

```

(b)

Figure A.1 Input for FORTRAN program CRACK. The first file (a) contains the specimen parameters and the second file (b) holds the rain-flow matrix.

1021

1.000	0.413	0.951	0.462	0.951	0.462	1.000	0.413	1.000	0.413
1.000	0.413	1.000	0.364	0.902	0.462	0.902	0.462	0.902	0.462
0.902	0.462	0.951	0.462	0.951	0.462	0.951	0.413	0.951	0.462
0.902	0.462	0.902	0.462	0.902	0.413	1.000	0.364	0.951	0.364
0.902	0.462	0.902	0.462	0.902	0.462	0.951	0.364	0.902	0.364
0.902	0.364	0.902	0.364	0.902	0.364	0.902	0.364	0.902	0.364
0.902	0.364	0.902	0.364	0.902	0.364	0.902	0.364	0.902	0.364
0.902	0.315	0.951	0.315	0.951	0.266	0.804	0.266	0.804	0.169
0.853	0.315	0.804	0.315	0.804	0.315	0.804	0.315	0.804	0.315
0.804	0.315	0.804	0.315	0.804	0.315	0.804	0.315	0.755	0.022
0.658	-0.271	0.315	-0.223	0.413	-0.125	0.462	0.022	0.658	0.071
0.609	-0.076	0.462	-0.027	0.707	-0.027	0.609	-0.027	0.511	0.022
0.609	-0.125	0.462	-0.027	0.609	0.169	0.609	0.169	0.609	0.169
0.804	0.169	0.658	0.169	0.658	0.169	0.609	0.169	0.609	-0.076
0.364	-0.076	0.364	-0.125	0.364	-0.125	0.413	-0.125	0.413	-0.174
0.266	-0.174	0.266	-0.223	0.609	-0.076	0.707	0.022	0.755	0.266
0.755	0.266	0.755	0.266	0.755	0.266	0.755	0.266	0.755	0.266
0.755	0.266	0.755	0.169	0.755	0.169	0.853	0.218	0.755	0.218
0.755	0.218	0.755	0.120	0.707	0.120	0.707	0.120	0.707	0.120
0.707	0.071	0.511	0.071	0.511	0.071	0.511	-0.027	0.755	0.315
0.755	0.315	0.755	0.315	0.804	0.364	0.804	0.364	0.951	0.462
0.951	0.462	0.951	0.413	0.951	0.413	0.951	0.413	0.853	0.413
0.853	0.413	0.853	0.413	0.853	0.413	0.853	0.364	0.951	0.413
0.951	0.413	1.000	0.413	0.951	0.413	0.951	0.413	0.951	0.413
0.951	0.413	0.951	0.413	0.951	0.413	0.951	0.413	0.951	0.413
0.951	0.413	0.951	-0.027	0.707	0.218	0.707	0.218	0.707	0.218
0.707	0.218	0.707	0.218	0.707	0.218	0.707	0.218	0.707	0.218
0.707	0.218	0.707	0.218	0.707	0.218	0.707	0.218	0.902	0.364
0.853	0.364	0.853	0.364	0.853	0.364	0.853	0.364	0.853	0.364
0.853	0.364	0.853	0.364	0.853	0.364	0.804	0.364	0.804	0.364
0.804	0.364	0.804	0.364	0.804	0.218	0.853	0.071	0.658	0.071
0.658	-0.027	0.560	0.120	0.560	0.120	0.560	0.120	0.560	0.120
0.609	0.120	0.609	0.120	0.658	-0.027	0.462	-0.223	0.315	-0.125
0.315	-0.125	0.462	-0.076	0.462	-0.076	0.462	-0.076	0.462	-0.076
0.511	-0.027	0.511	-0.027	0.511	-0.027	0.511	-0.027	0.511	-0.076
0.511	-0.076	0.413	-0.076	0.413	-0.076	0.413	-0.076	0.413	-0.076
0.413	-0.125	0.364	-0.076	0.364	-0.076	0.413	-0.076	0.413	-0.076
0.707	-0.125	0.658	0.120	0.609	0.120	0.609	0.120	0.609	0.022
0.511	-0.125	0.364	-0.271	0.560	0.022	0.511	0.022	0.511	0.022
0.511	-0.076	0.511	-0.076	0.511	-0.027	0.462	-0.027	0.462	-0.125
0.462	-0.076	0.560	0.022	0.560	0.022	0.560	-0.125	0.560	0.022
0.560	0.022	0.609	0.022	0.609	0.022	0.609	0.022	0.804	0.315
0.804	0.315	0.804	0.315	0.804	0.315	0.804	0.315	0.804	0.315
0.804	0.315	0.804	0.315	0.804	0.315	0.804	0.315	0.902	0.315
0.853	0.315	0.853	0.266	0.707	0.266	0.707	0.218	0.707	0.218
0.707	0.218	0.707	0.218	0.707	0.218	0.707	0.218	0.707	0.169
0.755	0.169	0.755	0.169	0.755	0.169	0.755	0.169	0.658	0.169
0.658	0.169	0.658	0.022	0.511	-0.076	0.609	-0.076	0.609	0.022
0.462	0.022	0.462	-0.125	0.560	0.071	0.560	-0.027	0.560	-0.027
0.609	-0.076	0.462	-0.076	0.462	-0.076	0.462	-0.076	0.413	-0.076
0.413	-0.125	0.315	-0.125	0.315	-0.125	0.315	-0.516	0.315	-0.174
0.315	-0.174	0.364	-0.271	0.462	-0.027	0.511	0.071	0.511	0.022
0.511	-0.076	0.511	0.022	0.560	0.071	0.560	-0.027	0.413	-0.027
0.413	-0.027	0.413	-0.027	0.413	-0.076	0.413	-0.027	0.560	-0.076
0.560	-0.076	0.609	0.071	0.609	-0.076	0.609	0.071	0.609	0.071
0.658	0.169	0.658	0.120	0.658	0.120	0.658	0.169	0.658	0.169
0.658	0.169	0.707	0.169	0.707	0.120	0.707	0.120	0.609	0.169
0.707	0.169	0.707	0.169	0.755	0.218	0.755	0.218	0.755	0.266
0.707	0.218	0.755	0.120	0.755	0.120	0.755	0.120	0.804	0.364

Figure A.2 Reconstructed History from Rain-Flow Matrix. The values are output from JRECON and input into SPEC11.

0.804	0.364	0.804	0.266	0.804	0.266	0.804	0.266	0.804	0.315
0.755	0.169	0.755	0.315	0.804	0.169	0.804	0.315	0.755	0.071
0.658	0.120	0.658	0.120	0.804	0.169	0.853	0.413	0.853	0.364
0.853	0.364	0.853	0.364	0.853	0.364	0.853	0.315	0.853	0.315
0.853	0.120	0.804	0.169	0.707	0.169	0.707	0.169	0.609	0.169
0.707	0.169	0.707	0.022	0.804	0.315	0.804	0.315	0.804	0.315
0.804	0.315	0.853	0.315	0.853	0.315	0.853	0.315	0.853	0.315
0.853	0.364	0.853	0.364	0.853	0.364	0.853	0.364	0.853	0.364
0.853	0.413	0.853	0.413	0.853	0.413	0.853	0.413	0.902	0.364
0.902	0.364	0.902	0.364	0.902	0.364	0.902	0.364	0.902	0.364
0.902	0.364	0.951	0.364	0.951	0.364	0.951	0.364	0.951	0.315
0.951	0.364	0.853	0.364	0.853	0.364	0.853	0.364	0.853	0.315
0.902	0.413	0.902	0.413	0.902	0.413	0.902	0.413	0.902	0.413
0.902	0.413	0.902	0.413	0.902	0.413	0.902	0.413	0.902	0.413
0.951	0.413	0.902	0.413	0.902	0.413	0.902	0.413	0.902	0.413
0.902	0.413	0.902	0.413	0.902	0.413	0.902	0.413	0.902	0.413
0.902	0.413	0.902	0.413	0.902	0.413	0.902	0.413	0.902	0.413
0.902	0.315	0.902	0.120	0.560	0.022	0.609	0.071	0.560	-0.027
0.755	0.218	0.755	0.218	0.755	0.218	0.755	0.218	0.755	0.266
0.755	0.266	0.755	0.266	0.902	0.266	0.804	0.218	0.755	0.218
0.755	0.071	0.853	0.364	0.853	0.364	0.853	0.364	0.853	0.364
0.902	0.315	0.951	0.266	0.853	0.218	0.804	0.218	0.707	0.266
0.707	0.266	0.804	0.169	0.804	0.169	0.804	0.071	0.560	-0.125
0.364	-0.125	0.364	-0.223	0.462	0.022	0.462	0.022	0.560	0.022
0.560	0.022	0.804	0.218	0.804	0.218	0.804	0.218	0.804	0.218
0.951	-0.369	0.266	-0.223	0.266	-0.174	0.266	-0.174	0.609	-0.076
0.658	-0.125	0.511	-0.125	0.511	-0.125	0.315	-0.223	0.266	-0.223
0.266	-0.223	0.413	-0.125	0.413	-0.125	0.462	0.022	0.462	-0.125
0.511	-0.076	0.560	0.071	0.560	0.071	0.560	0.071	0.609	-0.271
0.315	-0.174	0.315	-0.174	0.560	-0.027	0.511	-0.027	0.511	-0.174
0.364	-0.125	0.364	-0.125	1.000	0.462	1.000	0.364	1.000	0.462
0.951	0.462	0.951	0.462	0.951	0.462	0.951	0.462	0.951	0.462
0.951	0.120	0.609	0.120	0.609	0.120	0.658	0.120	0.658	0.022
0.707	0.218	0.658	0.071	0.755	0.071	0.755	0.071	1.000	0.462
0.902	0.266	0.755	0.266	0.755	0.266	0.755	0.266	0.755	0.266
0.755	0.266	0.755	-0.027	0.560	-0.125	0.511	-0.223	0.364	-0.223
0.315	-0.174	0.315	-0.174	0.560	-0.125	0.560	-0.125	0.755	0.120
0.707	0.120	0.707	0.071	0.707	0.120	0.609	0.120	0.609	0.071
0.707	0.071	0.707	0.022	0.853	0.218	0.707	0.218	0.707	0.218
0.707	0.071	0.658	0.071	0.560	0.022	0.560	0.022	0.560	0.022
0.560	-0.076	0.462	-0.174	0.315	-0.174	0.364	-0.174	0.315	-0.174
0.315	-0.174	0.315	-0.174	0.266	-0.320	0.364	-0.223	0.364	-0.076
0.364	-0.076	0.364	-0.076	0.364	-0.076	0.413	-0.223	0.853	0.169
1.000									

Figure A.2, continued Reconstructed History from Rain-Flow Matrix. The values are output from JRECON and input into SPEC11.